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Performance, Inclusion and Excellence: An Index of Educational Achievements for PISA

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ABSTRACT

This paper proposes an elementary multidimensional index that summarizes three relevant aspects of the educational achievements, out of the data provided by the PISA Reports. The three aspects considered are: *performance*, *inclusiveness*, and *excellence*, and will be approximated by the average scores and the distribution of outcomes in the different levels of proficiency. The Index of Educational Achievements (IEA) is the geometric mean of the normalised values of those three variables. We analyse the distribution of the variables that approach those three aspects and the resulting index, relative to the corresponding average test scores of the OECD countries. This index provides a much wider discrimination power than the mere comparison of the average scores, as the distribution of the three variables is very different.

Keywords: Education, PISA, multidimensional measurement, performance, inclusiveness, excellence, OECD.

INTRODUCTION

This paper aims at providing an elementary indicator of educational performance, using PISA data, which goes beyond the consideration of the average test scores by incorporating other relevant aspects of the educational outcomes.

PISA is the acronym of the Programme for International Student Assessment (PISA). This is a study, coordinated by the OECD, which provides the broadest dataset for the evaluation of schoolchildren performance and the characteristics of their schooling and family environment. It is a triennial worldwide test of 15-year-old schoolchildren's scholastic performance. The aim the PISA is to test and compare schoolchildren's performance across the world, with a view to improving educational methods and outcomes. Students also are asked to answer a questionnaire on their personal background, their learning habits, their attitudes towards studying, and their engagement and motivation. As a result, PISA reports provide an extensive and extremely rich profile of knowledge and skills among 15-year-olds, as well as contextual indicators relating performance results to student and school characteristics. [1]

There are many aspects that can be analysed with the rich database that PISA provides, some of which are already carefully dealt with in the PISA reports. Our focus here is to find a way of synthesising some basic traits of the educational achievements of those countries participating in the PISA survey in the field of science, which is the main subject of the 2015 wave. To do so

[1] PISA surveys started in 2000 with the aim of evaluating the students' ability, about the end of compulsory education, in three different domains: reading, mathematics and science. Every period of assessment specialises in one particular domain, but it also tests the other two. The subject specialisation is rotated through each PISA wave. Therefore, although information for any of the domains is available in all waves, there are some differences in the pieces of information obtained for a specific domain in each occasion. The 2015 report has focused on science.

we elaborate a multidimensional evaluation index that integrates three dimensions that we deem essential: *performance*, *inclusiveness*, and *excellence*. This is a way of enriching the evaluation by incorporating some of the diversity that the countries exhibit regarding the distribution of outcomes into the different levels of proficiency.

The PISA establishes six levels of educational proficiency, parameterized in terms of the scores of the tests that students perform for each subject. The distribution of the students into those levels of competence provides rich information on the functioning of educational systems, which is not reflected in the average scores or the associated ranking of countries. A simple inspection of the data regarding the distribution of the students among those levels of competence shows that there is a large diversity, even between countries with similar average scores. Those differences are particularly relevant at the tails (the percentages of high and low performers). The outcome distributions on those levels of proficiency provide, therefore, important information on the structural features of the different educational systems. The OCDE pays an increasing attention to those outcome distributions (see for instance OCDE (2016, vol. I)); yet, it does not provide any concrete indicator that summarises it and permits a systematic comparison. The index we propose here incorporates part of that information into de determination of the educational achievements in a very simple way. [2]

It is worth noting that the OECD pinpoints levels 2 and 5 as the relevant goalposts to assess on the effectiveness of educational systems in getting minimal outcomes, on the one hand, and assessing high performance, on the other hand. We shall follow this convention here and will identify level 2 as a sort of poverty line for educational outcomes and level 5 as the threshold for excellence.

The Index we propose here has many features in common with the new Human Development Index (UNDP (2010)), even though it avoids some of its problems (see Herrero, Martínez & Villar (2010), (2011)). It is a variant of some ideas developed in Villar (2013), aimed at taking into account different aspects of the outcome distributions.

Let us recall here that building a multidimensional index always involves three critical decisions: (a) *The number and nature of the dimensions considered*; (b) *The choice of the variables that measure those dimensions*; and (c) *The selection of the aggregation formula*. Those decisions, that are neither easy nor independent, determine the picture we obtain concerning the performance of the different educational systems. Simplifications and compromises are inevitable in that endeavour. Our way of dealing with this measure tries to meet three specific requirements that we think important for an index of this sort: (i) *Simplicity*: an index easy to understand and easy to use; (ii) *Accessibility*: an index that can be calculated directly from the published PISA reports, without requiring statistical abilities; and (iii) *Flexibility*: an index that can be adjusted and modified by the user to incorporate particular concerns.

We propose here a three-dimensional index of educational achievements that incorporates the distribution of the students' outcomes into the different levels of proficiency, while keeping the simplicity of real-valued measures. Those dimensions are: *Performance, Inclusiveness and Excellence*. *Performance* refers to the overall achievement of the students in a given society and

[2] See Villar (2015) for a procedure to obtain a comparative of complete distributions in terms of a real valued indicator. Applications of a variant of this procedure can be found in Herrero & Villar (2013), (2014), Herrero, Méndez & Villar (2014), Villar (2014).

is captured by the mean test scores of the aspect under consideration (science in this case). *Inclusiveness* refers to the capacity of an educational system to provide a minimal knowledge to the young. We shall use the percentage of students with at least proficiency level 2 to approach this aspect. Excellence deals with the extent of high performance and it is associated with the fraction of students equal or above level 5. Those three dimensions are certainly important and will be treated here symmetrically (see however the discussion below).[3]

Needless to say, the notions of performance, inclusiveness and excellence are complex and admit a number of interpretations. We are using those notions here as labels rather than as sound philosophical concepts, in order to give a more intuitive content to the variables we actually use. Be as it may, in view of the differences in the distribution of students by levels of proficiency, this index provides a more accurate estimate of the differences between educational systems because the spread of mean values is (artificially) very small and thus hides relevant information.

The paper is organised as follows. Section 2 presents the variables that approximate performance, inclusiveness and excellence, focussing on the field of science, which is the main subject of the 2015 wave. Section 3 introduces the Index of Educational Achievements. We compare the picture that this index provides and the one derived from computing average scores. A few final comments in Section 4 close the paper.

A MULTIDIMENSIONAL APPROACH TO EDUCATIONAL ACHIEVEMENTS

Let us recall here that, besides the average scores, the PISA classifies the students into six (actually seven) categories that approach the different skills that the students achieve, operationalized in term of ranges of the scores obtained by the students. Table 1 describes the score intervals for those levels and the percentage of students within each level in the OECD as a whole.

Table 1: Proficiency Levels in Science (PISA 2015)

Proficiency levels	< 2	2	3	4	5+
Thresholds of the test scores	< 410	411 - 484	485 - 559	560 - 633	> 633
% of OECD students	23,5	25,4	25,6	17,8	7,6

We shall describe now nature of the variables that enter the Index of Educational Achievements.

Performance

One of the assets of the PISA report is that it provides a unified scoring system to evaluate the performance of 15-year-old students in very different countries. The units of those scores are set with respect to the values obtained in the 2000 wave of the report, by taking a value of 500

[3] Needless to say one can think of other dimensions that are also relevant and/or of different degrees of relevance for those dimensions. Note, however, that the more dimensions we consider the more dependent becomes the index on the specific aggregation process and, in particular, on the weights we attach to those dimensions. Moreover, most of those dimensions tend to be rather elusive when it comes to give a precise content to their meaning and to select the variables that can measure them.

for the average of the OECD Member States with a standard deviation of 100. We shall take the countries average test score as our measure of *performance*.

The variability of the average scores is relatively low, partly due to the protocol that defines the measurement units, with a coefficient of variation for the OECD of 0.055. Yet, the difference between top and bottom performers is huge: there are 122 score points of difference between Japan and Mexico, equivalent to three years of schooling.

Table 2 gives the values corresponding to the average score in science for the OECD countries. We normalise the values by setting the OECD mean equal to 1 in order to get an intuitive appraisal of the countries' differences.

Korea, Japan, Estonia and Finland are the countries with better performance while Mexico, Chile, Turkey and Greece occupy the lowest positions in the ranking. Sweden, Czech Republic and Spain represent the mean of the OECD countries (more detailed information on all participating countries for this and the following variables is provided in the Appendix).

Table 2: Performance in the OECD countries in science (PISA 2015) (OECD mean = 1)

Countries	Average scores
Australia	1,034
Austria	1,004
Belgium	1,018
Canada	1,070
Chile	0,907
Czech Republic	1,000
Denmark	1,018
Estonia	1,084
Finland	1,076
France	1,004
Germany	1,033
Greece	0,923
Hungary	0,967
Iceland	0,960
Ireland	1,019
Israel	0,946
Italy	0,975
Japan	1,092
Korea	1,046
Latvia	0,994
Luxembourg	0,979
Mexico	0,843
Netherlands	1,032
New Zealand	1,041
Norway	1,011
Poland	1,017
Portugal	1,016
Slovak Republic	0,935
Slovenia	1,040
Spain	1,000
Sweden	1,001
Switzerland	1,025
Turkey	0,863
United Kingdom	1,033
United States	1,007

Inclusiveness

Inclusiveness is a dimension that refers to the capacity of an educational system to guarantee a minimum level of knowledge to all students. We take level 2 of proficiency to identify the students with sufficient knowledge to have good chances of integration in the labour market and social life. As mentioned before, the PISA takes precisely this level as the baseline of proficiency at which students begin to demonstrate the skills that will enable them to participate effectively and productively in life.[4]

According to the PISA 2015 report (vol. I, p. 60), "At Level 2, students are able to draw on everyday content knowledge and basic procedural knowledge to identify an appropriate scientific explanation, interpret data, and identify the question being addressed in a simple experimental design. They can use basic or everyday scientific knowledge to identify a valid conclusion from a simple data set. Level 2 students demonstrate basic epistemic knowledge by being able to identify questions that can be investigated scientifically." There is evidence, particularly longitudinal studies developed in Australia, Canada, Denmark and Switzerland, showing that students who perform below Level 2 often face severe disadvantages in their transition into higher education and the labour force in subsequent years. Consequently, "the proportion of students who perform below this baseline proficiency level thus indicates the degree of difficulty countries face in providing their populations with a minimum level of competencies." (cf. OECD (2014, vol. I, p. 68)).

We can think of the fraction of students below Level 2 as a measure of *educational poverty* (a simple head count ratio). Its complement, the fraction of the students with level of proficiency equal or above level 2, can therefore be regarded as a measure of inclusiveness. Note that this variable, simple as it is, involves equity and efficiency features. From an equity perspective this variable tells us about the share of students who are not under the educational poverty threshold. The equity significance of this measure is enhanced by the well-established correlation between educational outcomes and family background. From an efficiency viewpoint, this variable tells us the percentage of 15 year-old students who succeed in getting a minimal stock of human capital, provided by the educational system.

The diversity of educational systems regarding inclusion is large, with a coefficient of variation that is twice that of the average scores the OECD countries (note though that the CV of the students below level 2 is much larger, about eight times that of the average scores).

The most inclusive countries within the OECD are Estonia and Japan, with more than 90% of the population above level 2. Mexico and Turkey are those with lower values, even if we disregard the fact that a large proportion of their 15-year old people have already left the school. Table 3 below shows the normalised values of this variable for the OECD countries.

[4] This convention is not universal, though. Some authors adopt level 3 as the baseline (e.g. Nonoyama-Tarumi & Willms (2010).

Table 3: Inclusiveness in the OECD Countries in Science (PISA 2015)

(Mean OECD = 1)

Countries	Inclusiveness
Australia	1,046
Austria	1,005
Belgium	1,018
Canada	1,128
Chile	0,827
Czech Republic	1,006
Denmark	1,067
Estonia	1,157
Finland	1,123
France	0,989
Germany	1,053
Greece	0,854
Hungary	0,939
Iceland	0,948
Ireland	1,075
Israel	0,871
Italy	0,975
Japan	1,147
Korea	1,086
Latvia	1,051
Luxembourg	0,940
Mexico	0,662
Netherlands	1,034
New Zealand	1,048
Norway	1,032
Poland	1,062
Portugal	1,048
Slovak Republic	0,879
Slovenia	1,079
Spain	1,037
Sweden	0,995
Switzerland	1,034
Turkey	0,704
United Kingdom	1,048
United States	1,011

Excellence

Excellence is a dimension that captures the extent of high performance in the population of 15-year old students. It refers to the thickness of the right hand tail of the distribution of the students into the different levels of proficiency. The underlying idea is that the fraction of students with high levels of performance is a predictor of those who will get higher education and more likely to exert social leadership.

One of the simplest ways of having a proxy of the quality of an educational system is taking the share of the students in the upper levels of proficiency (levels 5 and 6 of the PISA study –see Table 1 above-). According to PISA: "At Level 5, students can use abstract scientific ideas or concepts to explain unfamiliar and more complex phenomena, events and processes involving multiple causal links. They are able to apply more sophisticated epistemic knowledge to evaluate alternative experimental designs and justify their choices and use theoretical knowledge to interpret information or make predictions. Level 5 students can evaluate ways of exploring a given question scientifically and identify limitations in interpretations of data sets including sources and the effects of uncertainty in scientific data. (OECD (2016), vol. I, p. 60)).[5]

We take as a measure of excellence the share of the students in levels of competence 5 and 6. This variable exhibits an extremely high variability, with a coefficient of variation of 0.5, nine times the coefficient of variation of the average scoring for the OECD countries. The countries with best outcomes regarding excellence are Japan and Finland, with more than 14% of its population in or above level 5. In the opposite side we find Mexico and Turkey, with less than 1% of its population in that category. Table 4 shows the data for the OECD countries with respect to this variable.

[5] Concerning level 6, PISA specifies: "At Level 6, students can draw on a range of interrelated scientific ideas and concepts from the physical, life and earth and space sciences and use content, procedural and epistemic knowledge in order to offer explanatory hypotheses of novel scientific phenomena, events and processes or to make predictions. In interpreting data and evidence, they are able to discriminate between relevant and irrelevant information and can draw on knowledge external to the normal school curriculum. They can distinguish between arguments that are based on scientific evidence and theory and those based on other considerations. Level 6 students can evaluate competing designs of complex experiments, field studies or simulations and justify their choices." (Ibid).

Table 4: Excellence in Science at the OECD Countries (PISA 2015)

(OECD mean = 1)

Countries	Excellence
Australia	1,455
Austria	1,000
Belgium	1,169
Canada	1,610
Chile	0,156
Czech Republic	0,948
Denmark	0,909
Estonia	1,753
Finland	1,857
France	1,039
Germany	1,377
Greece	0,273
Hungary	0,597
Iceland	0,494
Ireland	0,922
Israel	0,753
Italy	0,532
Japan	1,987
Korea	1,377
Latvia	0,494
Luxembourg	0,896
Mexico	0,013
Netherlands	1,442
New Zealand	1,662
Norway	1,039
Poland	0,948
Portugal	0,961
Slovak Republic	0,468
Slovenia	1,377
Spain	0,649
Sweden	1,104
Switzerland	1,273
Turkey	0,039
United Kingdom	1,416
United States	1,104

THE INDEX OF EDUCATIONAL ACHIEVEMENTS (IEA)

The Index of Educational Achievements, IEA for short, is a summary measure of the educational attainments in the three specific dimensions presented above: performance, inclusiveness and excellence. The index consists of the geometric mean of the normalized values of the variables that approximate those dimensions.

Consider a society *j* (typically a country, but also a region or even a school) in a given period (year 2015 in our case) and suppose we have the relevant data on the variables that approach those three dimensions for of the target population. Let P_j, I_j, E_j denote those values. The Index of Educational Achievements is given by:

$$IEA_j\left(P_j, I_j, E_j\right) = \left(\frac{P_j}{P_0} \times \frac{I_j}{i_0} \times \frac{E_j}{E_0}\right)^{1/3} = \sqrt[3]{p_j \times i_j \times e_j}$$

where P_0, I_0, E_0 are some reference values selected so that the resulting normalized variables, *p*, *i*, *e*, are easy to interpret.

The normalization is convenient in order to set the values of the different dimensions into some type of common units. We take here as reference values *the OECD average of each variable*, so that one hundred times *x* tells us the percentage of the OECD average that this variable represents, for *x = p, i, e*. All normalised variables represent, therefore, shares on the OECD average and their values are easily understandable. This normalisation has the convenient property that a change of the reference values will affect neither the ranking that the IEA produces among the different countries nor the relative valuations of any two societies.

The geometric mean is a centrality measure that exhibits better properties than the arithmetic mean, as it is an aggregator that penalises the dispersion of its components. That is, getting high values of the index requires doing well in all the three dimensions. Moreover, the geometric mean is a common centrality measure with an intuitive interpretation that many people can understand.[6]

The overall educational achievements of the OECD countries, as measured by the Index of Educational Achievements, exhibit much more variability than the average PISA scores. The IEA has a coefficient of variation of about five times that of the average scores (0.26 with respect to 0.055).

The different behaviour of the average test scores (performance) and the IEA, in the OECD countries, is well illustrated in Figure 1, where we present the shape of the distribution, ordering the countries according to their performance values (from top to bottom). Figure 1 points out two main aspects. First, that the variability is much higher for the EDI. Second, that there are also many changes in the ranking (that appear as peaks in the EDI line).

[6] The geometric mean can be characterized in terms of reasonable assumptions that adjust well to this context. See for instance Foster et al (2005), Seth (2009, 2010), or Herrero, Martínez & Villar (2010, 2011).

Figure 1: Performance and IEA in the OECD Countries (Science, PISA 2015)

The last column of Table 5 provides the values of the IEA for the OECD countries. The other columns describe the position that each country occupies in the ranking of the individual variables that conform the index as well as the ranking of the IEA. We observe that there are countries that exhibit very different positions in the ranking of the constituent variables (e.g. Denmark and Sweden), while others keep a rather homogeneous ranking.

Table 5: Index of Educational Achievements for the OECD (mean OECD = 1) and Ranking of the Constituent Variables

	Ranking Performance	Ranking Inclusion	Ranking Excellence	Ranking IEA	IEA score
Australia	8	15	6	6	1,163
Austria	20	23	18	21	1,003
Belgium	14	20	13	13	1,066
Canada	4	3	5	4	1,248
Chile	33	33	33	33	0,489
Czech Republic	23	22	20	23	0,984
Denmark	15	8	23	22	0,996
Estonia	2	1	3	3	1,300
Finland	3	4	2	2	1,309
France	21	25	16	17	1,010
Germany	10	10	9	11	1,144
Greece	32	32	32	32	0,599
Hungary	28	29	27	27	0,816
Iceland	29	27	29	30	0,766
Ireland	13	7	22	20	1,003
Israel	30	31	25	26	0,853
Italy	27	26	28	29	0,797
Japan	1	2	1	1	1,355
Korea	5	5	10	7	1,161
Latvia	25	11	30	28	0,802
Luxembourg	26	28	24	24	0,938
Mexico	35	35	35	35	0,194
Netherlands	11	17	7	9	1,154
New Zealand	6	12	4	5	1,220
Norway	18	19	17	16	1,027
Poland	16	9	21	18	1,008
Portugal	17	13	19	19	1,008
Slovak Republic	31	30	31	31	0,727
Slovenia	7	6	11	8	1,156
Spain	24	16	26	25	0,876
Sweden	22	24	14	15	1,032
Switzerland	12	18	12	12	1,105
Turkey	34	34	34	34	0,287
United Kingdom	9	14	8	10	1,153
United States	19	21	15	14	1,040

These data show that the IEA allows uncovering relevant differences among the countries with respect to some key features of the educational systems. Inclusiveness and Excellence exhibit patterns of behaviour that can be very different in pair-wise comparisons and shed light on the nature of the differential achievements. Indeed, the analysis of the distribution of the different variables is worth on its own, as it provides valuable information on the differences of the countries with respect to in the selected dimensions.

If we compute the *correlation* between each pair of variables within the IEA we observe that they are always positive and very high. Table 6 below provides the corresponding coefficients of correlation. We observe that the highest coefficient corresponds to that relating performance and inclusiveness. This points out, once more, that reducing scholastic failure is the main way of improving average scores (e.g. Willms (2006)). This is important because it implies that improving inclusiveness is not only an equity measure but also an efficiency one. Performance and excellence have also a very high correlation, most notably in OECD countries. The correlation between inclusiveness and excellence is also very high indicating that there is no trade-off between both variables. That is, getting a more inclusive society does not imply renouncing to the excellence; on the contrary, both variables go hand in hand together.

Table 6: Coefficients of Correlation between the IEA Components (Science, PISA 2015)

	Performance /Inclusiveness	Performance /Excellence	Inclusiveness /Excellence
OECD	0,978	0,933	0,840

FINAL COMMENTS

We have presented in this paper a proposal to evaluate the educational achievements of the countries, out of the data in the PISA, aimed at incorporating some relevant aspects that are not captured by the average test scores (inclusiveness and excellence). The Index of Educational Achievements (IEA) is simply the geometric mean of the normalized values of the variables that approach those dimensions. The data show that the information provided by this construct helps evaluating systematically the students' results with a much large discrimination power. This may help policy makers identifying the key targets to look for improvements.

We have mentioned in the Introduction that we aimed at the construction of an index *easy* to handle and interpret, *accessible*, and *flexible*. The formula of the index and the choice of variables that measure the different dimensions ensure the first of these constraints. Accessibility is guaranteed by the OECD handy webpage, which offers the data in a readily usable format (standard spread-sheets). Flexibility refers to the possibility that any interested person, not necessarily a specialist, can perform alternative evaluation exercises by simple manipulations of those data (much in the spirit of what is proposed in the OECD Better Life Index).

There are four specific concerns, regarding flexibility, that this construction permits handling. The first one is the possibility of using different weights for the different dimensions. That is, people may have different perceptions about how important is performance, inclusiveness and excellence, and may be willing to know how countries fare under different configurations of

those weights. This can be easily done using differential powers, to express our judgement of the relevance of those aspects. That is, using the formula:

$$IEA = p^{\alpha} \times i^{\beta} \times e^{\gamma}$$
$$\text{with } \alpha + \beta + \gamma = 1.$$

The second one deals with the incorporation of additional dimensions or the use of composite ones. This applies in particular to using not only the results in science (*s*) but also those in reading literacy (*r*) and mathematics (*m*). Given the properties of the geometric mean, this can be achieved by simply taking the geometric mean of the corresponding geometric means. That is:

$$IEA = \sqrt[3]{\left(p_m \times p_r \times p_s\right)^{1/3} \times \left(i_m \times i_r \times i_s\right)^{1/3} \times \left(e_m \times e_r \times e_s\right)^{1/3}}$$

(and, of course, one can combine this generalisation with that consisting of given different weights to the different dimensions discussed above).

The third one refers to the very nature of the index. The geometric mean can be replaced by the arithmetic mean when the disparities between the (normalised) variables is deemed irrelevant or when there are countries for which some variable is very close to zero (as it happens with excellence in some Partner countries).

The last concern involves the adjustment of the index by the shares of students attending school. The PISA results evaluate the knowledge of those students *attending* school, rather than that of 15-year old people. This difference turns out to be very important in some countries in which more than a quarter of the 15-year old population has abandoned the educational system and are, therefore, out of the sample (Mexico and Turkey in particular, within the OECD). As there is no information on that part of the population it is not clear how to adjust the data to take into account those different participation rates. A simple (yet arbitrary) way of introducing that element into our analysis would be to add ½ of the young who have abandoned the school at 15 to the population below level 2. In the absence of information on this population, assuming that one half have not reached level 2 is a prudent proposal. This change in the IEA can be easily done out the tables provided in the PISA report.

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APPENDIX: THE IEA AND ITS COMPONENTS FOR PARTNER COUNTRIES

We present here the data corresponding to the Partner countries in two different formats, which may be of interest. Table A.1 contains the IEA and its components, elaborated so that they are fully comparable with the OECD results. Note that there appear some values equal to zero due to the negligible fraction of students with proficiency equal or above 5. A zero in one of the variables for a given country drives its evaluation to zero, no matter how it does in the other dimensions, due to the multiplicative nature of the formula. Table A.2 also contains information on the Index of Educational Achievements and its components, but now the values have been normalised with respect to the mean of each variable within this group of countries, and denoted by IEA*. Note that those values are not comparable with those of the OECD any more. Besides, we also provide the arithmetic mean of those normalised variable in order to avoid the problem of the zeros. This amounts to ignoring the disparities in the variables that compose the index for each country.

Table A.1: The IEA and its Components for Partner Countries

	Performance	Inclusion	Excellence	IEA
Albania	0,867	0,740	0,052	0,322
Algeria	0,762	0,371	0,000	0,000
Brazil	0,813	0,551	0,091	0,344
B-S-J-G (China)	1,050	1,063	1,766	1,254
Bulgaria	0,904	0,788	0,377	0,645
Chinese Taipei	1,080	1,112	2,000	1,339
Colombia	0,843	0,647	0,052	0,305
Costa Rica	0,851	0,680	0,013	0,196
Croatia	0,964	0,957	0,506	0,776
Cyprus[2]	0,877	0,735	0,208	0,512
Dominican Republic	0,673	0,181	0,000	0,000
FYROM	0,778	0,471	0,026	0,212
Georgia	0,834	0,624	0,117	0,393
Hong Kong (China)	1,061	1,150	0,961	1,055
Indonesia	0,818	0,558	0,013	0,181
Jordan	0,829	0,637	0,026	0,239
Kosovo	0,768	0,410	0,000	0,000
Lebanon	0,784	0,475	0,052	0,268
Lithuania	0,964	0,956	0,545	0,795
Macao (China)	1,072	1,166	1,195	1,143
Malta	0,943	0,857	0,987	0,927
Moldova	0,868	0,734	0,091	0,387
Montenegro	0,834	0,622	0,065	0,323
Peru	0,805	0,527	0,013	0,177
Qatar	0,847	0,637	0,221	0,492
Romania	0,882	0,780	0,091	0,397
Russia	0,987	1,038	0,481	0,790
Singapore	1,127	1,147	3,143	1,596
Thailand	0,855	0,676	0,065	0,335
Trinidad and Tobago	0,861	0,688	0,182	0,476
Tunisia	0,784	0,433	0,000	0,000
United Arab Emirates	0,886	0,739	0,364	0,620
Uruguay	0,883	0,751	0,169	0,482
Viet Nam	1,064	1,194	1,078	1,111

Table A.2: The IEA* and its Components for Partner Countries

	Performance	Inclusion	Excellence	Arithmetic mean	IEA*
Albania	0,975	1,005	0,133	0,705	0,508
Algeria	0,858	0,503	0,000	0,454	0,000
Brazil	0,915	0,748	0,233	0,632	0,543
B-S-J-G (China)	1,182	1,445	4,533	2,387	1,978
Bulgaria	1,018	1,071	0,967	1,018	1,017
Chinese Taipei	1,215	1,510	5,133	2,620	2,112
Colombia	0,949	0,879	0,133	0,654	0,481
Costa Rica	0,958	0,924	0,033	0,638	0,309
Croatia	1,085	1,300	1,300	1,228	1,224
Cyprus[2]	0,988	0,998	0,533	0,840	0,807
Dominican Republic	0,757	0,247	0,000	0,335	0,000
FYROM	0,876	0,640	0,067	0,527	0,334
Georgia	0,939	0,848	0,300	0,696	0,620
Hong Kong (China)	1,195	1,562	2,467	1,741	1,664
Indonesia	0,920	0,759	0,033	0,571	0,286
Jordan	0,933	0,866	0,067	0,622	0,378
Kosovo	0,864	0,557	0,000	0,474	0,000
Lebanon	0,882	0,645	0,133	0,554	0,423
Lithuania	1,085	1,298	1,400	1,261	1,254
Macao (China)	1,207	1,584	3,067	1,953	1,803
Malta	1,061	1,164	2,533	1,586	1,463
Moldova	0,977	0,997	0,233	0,736	0,610
Montenegro	0,939	0,845	0,167	0,650	0,509
Peru	0,906	0,716	0,033	0,552	0,278
Qatar	0,953	0,866	0,567	0,795	0,776
Romania	0,993	1,060	0,233	0,762	0,626
Russia	1,111	1,410	1,233	1,252	1,246
Singapore	1,268	1,559	8,067	3,631	2,517
Thailand	0,962	0,919	0,167	0,683	0,528
Trinidad and Tobago	0,969	0,934	0,467	0,790	0,751
Tunisia	0,882	0,588	0,000	0,490	0,000
United Arab Emirates	0,997	1,003	0,933	0,978	0,977
Uruguay	0,994	1,021	0,433	0,816	0,760
Viet Nam	1,198	1,622	2,767	1,862	1,752

