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Softness Parameter and the Energy Ratios for Deformed Nuclei

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ABSTRACT
In this article the softness parameter is calculated for the chosen nuclei Using the NS2 model. The energies are calculated from NS2 model, and AB model. The experimental energy ratios (R4) and predicted energy ratios from NS2 model are calculated. The predicted values of energies and energy ratios are compared with experimental data.

Key words: Rotational bands; Softness model, angular momentum, Softness parameter

INTRODUCTION

The energy levels of ground bands of deformed nuclei are written as [1]

$$E(I) = \frac{\hbar^2}{2\theta} I\left(I+1\right) \qquad (1)$$

Where θ is the moment of inertia, and I is the spin, takes the values $I^p = 0^+, 2^+, 4^+, 6^+$,--

The experimental data shows deviation from predicted data of Equation (1).

There are many attempts to develop Eq.(1) , Mariscotti et al [2] proposed the variable moment of inertia model (VMI), where the excitation energy of the ground band is described as:

$$E(I) = \frac{I(I+1)}{\theta(I)} + \frac{1}{2}C\{\theta(I) - \theta_0\}^2 \qquad (2)$$

Where C and q_0 are two parameters
The an harmonic vibrator model (AVM) [3] is written as

$$E(I) = AI + \frac{I(I-2)}{\theta(I)} + \frac{1}{2}C\{\theta(I) - \theta_0\}^2 \qquad (3)$$

Where A, C and q_0 are fitting parameters.
Klein et al proposed GVMI [4, 5, 6]

$$E(I) = \frac{I + XI(I-2)}{\theta(I)} + \frac{1}{2}K(\theta(I) - \theta_0)^2 \qquad (4)$$

It is clear that at X=1/3 Eq.(4) becomes VMI model (Eq.(2))

R.K. Gupta et al [7, 8, 9] ;proposed the deviation of observed data from two parameter **I (I+1)** expansion , may be attributed due to the change of moment of inertia. In this work, the effect of variation of moment of inertia is considered in Eq.(1) to formulate "NS2 MODEL" which describing energies of states in rotation ground bands nuclei **.** The predicted results of NS2 model and AB model are compared with experimental data. The energy ratio R4 is calculated from experimental data and NS2 model. Also, the energy ratios R(I)=E(I)/E(2) are calculated by NS2 model .

METHOD AND RESULTS

We know that the ground state bands of deformed nuclei are described by the formula [1]:

$$E(I)=\frac{\hbar^2}{2\theta_0}I(I+1) \qquad (1)$$

Introducing the effect of rotation – vibration interaction [6, 7], we obtain
The energy as:

$$E(I)=\frac{\hbar^2}{2\theta_0}I(I+1) \quad +B[I(I+!]^2 \qquad (5)$$

$$= AI(I+1) \quad +B[I(I+!]^2$$

where A and B are fitting parameters

According to R.K .Gupta [8, 9, 10] which introduce the concept of variation of moment of inertia with angular momentum

i.e. $$\theta(I)=\theta_0(1+o_1I+o_2I^2+o_3I^3+---+---) \qquad (6)$$

Where θ_0 is the moment of inertia at I=0, and σ_n is the softness parameter

$$o_n=\frac{1}{n!}\frac{\delta^n\theta(I)}{\delta I^n}|_{I=0} \qquad (7)$$

$n = 1, 2,3, \ldots$

Take n=1, then

$$\theta(I)=\theta_0(1+o_1I) = \qquad (8)$$

or $\theta(I)=\theta_0(1+oI)$

Substitute in Eq.(1) one gets

$$E(I)=\frac{\hbar^2}{2\theta_0(1+o_1I)}I(I+1)$$

$$E(I) = \frac{\hbar^2}{2\theta_0(1+o_1 I)} I(I+1) = \frac{\hbar^2}{2\theta_0} I(I+1)(1+o_1 I)^{-1}$$

$$= \alpha I(I+1) - \alpha\sigma_1 I^2 (I+1) + O \quad ,$$

$$where \ \alpha = \frac{\hbar^2}{2\theta_0} \ , \sigma_1 \ is\, the \ softness \ parameter \ and \ O \ is\, a \ small \ quantity$$

Then one can write

$$E(I) = \alpha I(I+1) - \alpha\sigma_1 I^2 (I+1) \ , \qquad (9)$$

One calculated the energy levels of the ground state bands for considering nuclei ,and the energy ratios R4=E(4)/E(2) using the experimental and predicted energy values of NS2 model. Also the energy ratios R(I)=E(I)/E(2) are calculated by NS2 model .The predicted energy of NS2 model and AB model are compared with the experimental data

RESULTS AND DISCUSSION

Using the least square fitting, and the experimental energies, the parameters A, and B , for AB model Eq. (5) ,and the parameters a and s_1 for NS2 model as in Eq.(9). The parameters are listed in Table (1) for considering nuclei. The first column specified the nucleus the second and third columns for A &B parameters, The fourth and fifth columns for the a and s_1 parameters and the last one for experimental energy ratio R4=$E(4)_{esp.}/E(2)_{exp}$.

The predicted energies for chosen nuclei according to NS2 model and AB model are listed in table (2). The first column named the nucleus ,the second the spin I, the third for experimental energy ($E_{exp.}$) ,the fourth one for predicted energy of NS2 model ,the fifth for predicted energy of AB model (E_{AB}), and the last column for the predicted energy ratio ($R=E_{NS2}(I)/E_{NS2}(2)$) by NS2 model . The deviation from experimental data is calculated as $\tau = \frac{1}{N}\sum_{i=1}^{N}\left(E_{Cal} - E_{exp}\right)$ .It is tabulated in the last raw for each nucleus, where $E_{cal.}$ represent predicted energy, E_{exp}. Is experimental energy and N is the number of used data.

CONCLUSION

It is clear that the two models (AB &NS2) give a good agreement with the experimental data in this range of spin. Fig.1.a. represent the sigma σ_1 versus R4 (calculated from experimental data.) and Fig.1.b. Represent σ_1 versus R4 (as calculated from NS2 in Eq. (9)). The two figures show scatter shape but gross trend is decrease of σ_1 with increasing R4.This means that the softness tends to zero as the nucleus becomes rigid. Also the relative ratios R (I) is increase with increasing I.

Table (1): The fitting parameters of AB &NS2 models (Eq. (5) & Eq.(9)) and R4

Nucleus	Parameters AB model		Parameters of NS2		$E_{exp}(4)/E_{exp}(2)$
	A	B	a	s_1	R4
Zn62	9.502735E-02	-2.241677E-04	.1155424	3.987516E-02	2.292514
Ge66	9.597801E-02	-3.045714E-04	.1235022	0.0504203	2.271117
Se74	5.537131E-02	-1.321831E-04	.0681659	4.081554E-02	2.05766
Kr74	4.088962E-02	-5.647274E-05	4.732154E-02	2.703448E-02	2.224054
Kr78	4.662265E-02	-8.103195E-05	5.411156E-02	3.090469E-02	2.440165
Xe118	3.529722E-02	-8.084181E-05	4.277107E-02	3.901264E-02	2.401839
Xe120	3.431273E-02	-6.788295E-05	4.071381E-02	3.470313E-02	2.468672
Xe122	3.757673E-02	-8.538358E-05	4.527276E-02	3.851753E-02	2.501359
Xe 124	4.029112E-02	-1.000888E-04	4.920323E-02	4.133577E-02	2.482203
Xe126	.0426091	-1.090345E-04	5.234979E-02	4.238111E-02	2.424929
Sm150	3.244971E-02	-8.530622E-05	4.040544E-02	4.373793E-02	2.316215
Sm152	1.749752E-02	-2.430798E-05	1.967245E-02	2.515879E-02	3.009031
Dy154	3.097904E-02	-8.203716E-05	0386848	4.406592E-02	2.232516
Dy162	1.332269E-02	-7.253847E-06	1.394284E-02	1.039848E-02	3.293702
Yb158	3.581877E-02	-9.089389E-05	4.421196E-02	4.241284E-02	2.32952
Yb160	2.952434E-02	-6.851598E-05	.0356348	3.909619E-02	2.644628
Yb162	2.322436E-02	-4.131634E-05	2.689238E-02	3.118159E-02	2.92437
Yb164	1.892725E-02	-2.588343E-05	0211879	2.462856E-02	3.124899
Yb166	1.647693E-02	,1.166821E-05	1.507295E-02	1.800388E-02	3.227761
Pt182	.0192172	-3.19317E-05	2.212989E-02	2.961538E-02	2.671359
Pt184	2.016141E-02	-3.954459E-05	2.372676E-02	3.404245E-02	2.559269
Pt186	2.207444E-02	-4.358444E-05	2.606436E-02	3.437248E-02	2.524879
Pt188	3.188777E-02	-8.916165E-05	3.959438E-02	.0452079	2.819728
Os192	2.700998E-02	-4.207892E-05	3.083399E-02	2.796632E-02	2.896024
Th224	1.328431E-02	-2.193512E-05	.0152668	2.937657E-02	3.13615
Th226	1.106159E-02	-1.377572E-05	1.227402E-02	2.269814E-02	3.234526
Th228	9.289434E-03	9.289434E-03	1.004467E-02	1.743146E-02	3.272556
Th230	8.708182E-03	-6.243122E-06	9.245018E-03	1.352874E-02	3.283842
Th232	8,0210876E-03	-4,0965721E-06	8.596294E-03	1,2634E-02	3.283842
Th234	8.210876E-03	-4.965721E-06	8.62965E-03	1.143853E-02	3.277262
U230	8.475949E-03	-6.070965E-06	8.997855E-03	1.351573E-02	3.291222
U232	7.857332E-03	-4.72119E-06	8.259883E-03	1.141211E-02	3.295577
U234	7.202466E-03	-4.122788E-06	7.553479E-03	1.089123E-02	3.303921
U236	7.512401E-03	-3.575236E-06	7.815689E-03	9.114597E-03	3.3035
U238	7.450692E-03	-3.534543E-06	7.750933E-03	9.091041E-03	3.303832
Pu236	7.406847	-3.349731E-06	7.691239E-03	8.680746E-03	3.311371
Pu238	7.317455E-03	-2.537938E-06	7.53363E-03	6.723377E-03	3.308659
Pu240	7.11277E-03	-2.799443E-06	7.350647E-03	7.593435E-03	3.30714
Pu242	7.406967E-03	-2.933149E-06	7.655323E-03	7.628625E-03	3.30714
Pu242	7.687667E-03	-3.455534E-06	7.984657E-03	8.668354E-03	3.28

Table (2): Experimental and Theoretical Energies in (Mev) of ground bands nuclei experimental data are taken [11,12] .

Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Zn62	2	0.9538	0.637967	0.562094	1
	4	2.1866	1.942266	1.81088	3.044461547384112
	6	3.7078	3.691748	3.595717	5.786738185517433
	8	5.4815	5.665264	5.679884	8.880183457765057
	10	7.6	7.641665	7.74058	11.97815090749208
	12	9.4662	9.399801	9.368922	14.7339925105844
Dev.			0.4171881	0.1063038	
Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Ge66	2	0.95693	0.666289	0.564904	1
	4	2.1733	1.971883	1.797732	2.959501057348988
	6	3.564	3.617884	3.493812	5.42990203950538
	8	5.3584	5.305396	5.331518	7.962604815628053
	10	6.5021	6.735522	6.872266	10.10900975402566
	12	7.727	7.609365	7.560518	11.42051722300683
Dev.			0.3753903	0.1101633	
Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
	2	0.663477	0.375609	0.327469	1
Se74	4	1.36521	1.14074	1.054553	3.037041178459515
	6	2.2315	2.161846	2.092424	5.755575611872985
	8	3.1984	3.305381	3.301497	8.800058039077871
	10	4.2563	4.437798	4.491428	11.81494053656861
	12	5.5443	5.425549	5.421116	14.44467251849663
Dev.			0.4122642	9.511662E-02	
Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Kr 74	2	0.455623	0.268578	0.243305	1
	4	1.01333	0.844086	0.795203	3.142796506042937
	6	1.7814	1.665118	1.617746	6.199755750657165
	8	2.7479	2.670266	2.651298	9.94223651974473
	10	3.3892	3.798125	3.814538	14.14160876914714
	12	5.1796	4.987286	5.00446	18.56922756145328
Dev.			0.4122642	9.511552E-02	
Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Kr 78	2	0.424	0.304602	0.276819	1
	4	1.03463	0.948447	0.90004	3.113725451572872
	6	1.869083	1.851266	1.815211	6.077655432334653
	8	2.8787	2.932787	2.936761	9.628259170983776
	10	4.0679	4.112741	4.148005	13.50201574513628
	12	5.347	5.310856	5.301139	17.43539438349059
Dev.			0.1606141	4.055657E-02	
Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Xe118	2	0.3372	0.2366031	0.208873	1
	4	0.8099	0.7219324	0.6736076	3.051238128325453
	6	1.3964	1.375895	1.339878	5.815202759389036
	8	2.0729	2.118396	2.122316	8.953373814628802
	10	2.814	2.869344	2.904508	12.1272460081884
	12	3.588	3.548645	3.539	14.95753859522551
Dev.			0.147585	3.836955E-02	

Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Xe120	2	0.3224	0.227328	0.203433	1
	4	0.7959	0.701245	0.659101	3.08472779420045
	6	1.3971	1.35393	1.321389	5.955843538851351
	8	2.0489	2.117566	2.118611	9.315024985923423
	10	2.87	2.924333	2.953017	12.86393669059685
	12	3.749	3.706412	3.700786	16.30424760698198
Dev.			0.1524872	3.782713E-02	
Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Xe122	2	0.3311	0.250711	0.222387	1
	4	0.8282	0.765952	0.717381	3.05511924087894
	6	1.4667	1.46202	1.427606	5.831495227572783
	8	2.2173	2.255213	2.262896	8.995269453673752
	10	3.0389	3.061829	3.100299	12.21258341277407
	12	3.8195	3.798166	3.784075	15.14957859846596
Dev.			0.1078094	0.0311759	
Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Xe124	2	0.354	0.270813	0.238144	1
	4	0.8787	0.821356	0.765787	3.032926779733617
	6	1.5482	1.554005	1.51567	5.738295428949127
	8	2.3307	2.371133	2.3821	8.755609959640047
	10	3.1703	3.175117	3.220948	11.72438915413957
	12	3.8814	3.86833	3.849653	14.28413702444122
Dev.			0.1025456	3.183288E-02	
Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Xe126	2	0.3883	0.287475	0.251729	1
	4	0.9416	0.869505	0.808568	3.024628228541612
	6	1.6346	1.639593	1.597245	5.703428124184712
	8	2.4353	2.491247	2.50262	8.66596051830594
	10	3.317	3.317971	3.367683	11.54177232802852
	12	4.028	4.013269	3.993556	13.96041047047569
Dev.			0..1257393	3.723302E-02	
Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Sm 150	2	0.33395	0.221226	0.191627	1
	4	0.7735	0.666729	0.614872	3.013791326516775
	6	1.27885	1.251681	1.212408	5.657928995687668
	8	1.8371	1.891255	1.894152	8.548972543914368
	10	2.432	2.500623	2.537263	11.30347698733422
	12	3.043	2.994956	2.986143	13.53799282182022
Dev.			0.1719296	4.365571E-02	

Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Sm 152	2	0.1218	0.112096	0.10411	1
	4	0.3665	0.353854	0.340227	3.156704967170996
	6	0.7069	0.701519	0.692017	6.258198330002855
	8	1.1254	1.131334	1.133809	10.0925456751356
	10	1.6093	1.619541	1.630601	14.44780366828433
	12	2.1489	2.142384	2.138055	19.11204681701399
Dev.			1.807275E-02	6.663436E-03	

Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Gd 152	2	0.34428	0.212766	0.183514	1
	4	0.7554	0.639953	0.588404	3.0077784984443
	6	1.22729	1.198438	1.158728	5.632657473468505
	8	1.7467	1.805103	1.806577	8.483982403203519
	10	2.3004	2.376825	2.412073	11.17107526578495
	12	2.8837	2.830483	2.823373	13.26985044602991
Dev.			0.1942027	4.751682E-02	
Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Gd 154	2	0.1231	0.113849	0.105793	1
	4	0.371	0.359469	0.345749	3.157419037497036
	6	0.7177	0.712828	0.703323	6.261170497764583
	8	1.1445	1.149893	1.152516	10.10015898251192
	10	1.6372	1.64663	1.657876	14.46328030988414
	12	2.185	2.179008	2.174495	19.13945664871892
Dev.			1.682238E-02	6.458162E-03	
Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Dy154	2	0.3346	0.211653	0.182921	1
	4	0.747	0.637322	0.586766	3.011164500385064
	6	1.2241	1.195182	1.156406	5.646893736445975
	8	1.7483	1.803409	1.80521	8.520592668188025
	10	2.3059	2.380178	2.415045	11.245661530902
	12	2.8949	2.843665	2.836274	13.43550528459318
Dev.			0.1833915	4.536284E-02	
Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Dy162	2	0.08066	0.0819	0.079675	1
	4	0.26567	0.267258	0.263552	3.263223443223443
	6	0.54853	0.549063	0.546757	6.704065934065934
	8	0.92128	0.920373	0.92163	11.23776556776557
	10	1.37515	1.374229	1.377724	16.77935286935287
	12	1.903	1.903672	1.90181	23.24385836385836
Dev.			2.223313E-03	5.235374E-04	
Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Yb160	2	0.242	0.197091	0.174679	1
	4	0.64	0.601241	0.56308	3.050575622428218
	6	1.149	1.145579	1.11916	5.812436894632429
	8	1.738	1.763231	1.770566	8.946278622565211
	10	2.376	2.387325	2.418634	12.11280575977594
	12	2.963	2.950987	2.938392	14.9727131122172
Dev.			0.0625461	2.058149E-02	

Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Yb162	2	0.1666	0.151292	0.137859	1
	4	0.4872	0.470764	0.447961	3.111625201596912
	6	0.9239	0.918166	0.902541	6.068833778388811
	8	1.4452	1.453248	1.45797	9.605583903973773
	10	2.0236	2.03576	2.054752	9.605583903973773
	12	2.6335	2.625451	2.617526	17.35352827644555
Dev.			2.531891E-02	1.023175E-02	
Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Yb164	2	0.1233	0.120866	0.112632	1
	4	0.3853	0.382012	0.368192	3.160624162295435
	6	0.7602	0.758391	0.749286	6.274642993066702

	8	1.2233	1.224956	1.228582	10.1348269984942
	10	1.7537	1.756659	1.768808	14.53393841113299
	12	2.3303	2.328451	2.322751	19.2647311899128
Dev.			4.76525E-03	4.308322E-03	
Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Yb166	2	0.1023	0.0937	0.0993	1
	4	0.3302	0.323169	0.334206	3.44897545357524
	6	0.6679	0.70145	0.712614	7.486125933831377
	8	1.0978	1.241563	1.246827	13.25040554962647
	10	2.1757	1.956534	1.953647	20.88083244397012
	12	2.779	2.859388	2.854358	30.51641408751334
Dev.			2.289683E-02	8.005473E-03	
Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Pt182	2	0.1541	0.124915	0.114154	1
	4	0.4175	0.390167	0.371571	3.123459952767882
	6	0.7714	0.764298	0.750795	6.118544610335028
	8	1.2024	1.21585	1.218105	9.733418724732818
	10	1.695	1.713364	1.727519	13.71623904254893
	12	2.238	2.225382	2.220794	17.81517031581475
			4.442448E-02	1.257714E-02	
Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Pt 184	2	0.1634	0.132668	0.119545	1
	4	0.4365	0.409918	0.38741	3.089803117556607
	6	0.7981	0.792979	0.777022	5.977168571170139
	8	1.23	1.243082	1.246622	9.369870654566286
	10	1.705	1.721455	1.739265	12.97566104863268
	12	2.201	2.189328	2.182822	16.50230650948232
Dev.			4.456976E-02	1.355228E-02	
Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Pt186	2	0.1915	0.145635	0.130878	1
	4	0.4901	0.449616	0.424055	3.087279843444227
	6	0.8772	0.868937	0.850244	5.966539636763141
	8	1.3411	1.360598	1.363418	9.342520685274831
	10	1.8557	1.881593	1.900817	12.91992309540976
	12	2.407	2.388922	2.382942	16.40348817248601
Dev.			6.729883E-02	0.0183743	

Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Pt188	2	0.26589	0.216087	0.188117	1
	4	0.67134	0.648689	0.602091	3.0019806837061
	6	1.18427	1.211889	1.182005	5.608338308181427
	8	1.78225	1.819768	1.833705	8.421459874957772
	10	2.4371	2.386405	2.428799	11.04372313003559
	12	2.81007	2.825883	2.804654	13.077524330478
Dev.			0.0421982	1.859152E-02	
Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Os 192	2	0.2058	0.174656	0.160545	1
	4	0.5803	0.547695	0.523368	3.135849899230487
	6	1.0886	1.077725	1.060192	6.170558125687065
	8	1.7081	1.723355	1.726582	9.867138832905826
	10	2.4185	2.443195	2.461943	13.98861189996336
	12	3.212	3.195852	3.189525	18.29798002931477
Dev.			5.082202E-02	1.519085E-02	

Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Th224	2	0.0981	0.0862	7.89E-02	1
	4	0.2841	0.269457	0.256912	3.125951276102088
	6	0.5347	0.528187	0.519248	6.12745939675174
	8	0.8339	0.840881	0.842759	9.755
	10	1.1738	1.186013	1.19586	13.75885150812065
	12	1.5498	1.542054	1.53854	17.88925754060325
Dev.			2.158902E-02	7.027531E-03	

Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Th 226	2	0.0722	0.0703	0.065874	1
	4	0.22643	0.223193	0.215722	3.174864864864865
	6	0.4473	0.445302	0.440286	6.334310099573257
	8	0.7219	0.723257	0.725021	10.28815078236131
	10	1.0403	1.043685	1.050089	14.84615931721195
	12	1.3952	1.393213	1.390362	19.81810810810811
Dev.			4.379235E-03	2.662849E-03	

Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Th 228	2	0.057759	0.0582	5.54E-02	1
	4	0.186823	0.186886	0.182304	3.211099656357388
	6	0.378179	0.377753	0.37479	6.490601374570447
	8	0.6225	0.622363	0.623683	10.69352233676976
	10	0.9118	0.912311	0.916437	15.67544673539519
	12	1.2394	1.239194	1.237166	21.29199312714777
Dev.			2.122372E-04	1.109578E-03	

Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Th 230	2	0.0532	0.0540	5.20E-02	1
	4	0.1741	0.174895	0.171666	3.238796296296296
	6	0.3566	0.356772	0.354731	6.606888888888889
	8	0.5941	0.593599	0.594625	10.99257407407407
	10	0.8797	0.879371	0.882358	16.28464814814815
	12	1.2078	1.208085	1.206544	22.37194444444444
Dev.			1.191441E-03	5.919542E-04	

Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Th 232	2	0.049369	5.03E-02	4.86E-02	1
	4	0.16212	0.163236	0.160462	3.022888888888889
	6	0.3332	0.333671	0.331947	6.179092592592593
	8	0.5569	0.556365	0.55731	10.30305555555556
	10	0.827	0.826104	0.828725	15.29822222222222
	12	1.1371	1.137674	1.136274	21.06803703703704
Dev.			1.634829E-03	3.9603E-04	

Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Th 234	2	0.04955	0.0506	4.91E-02	1
	4	0.163	0.164696	0.162231	3.254861660079051
	6	0.3365	0.33757	0.336097	6.671343873517787
	8	0.5648	0.564478	0.565441	11.15569169960474
	10	0.843	0.84068	0.843111	16.61422924901186
	12	1.1602	1.161439	1.160051	22.95333992094862
Dev.			2.406575E-03	1.720358E-04	

Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
U 230	2	0.05172	0.05.25	5.06E-02	1
	4	0.1695	0.170228	0.167091	3.242438095238095

	6	0.3471	0.347264	0.345281	6.614552380952381
	8	0.5782	0.577797	0.578796	11.00565714285714
	10	0.8564	0.85599	0.858896	16.30457142857143
	12	1.1757	1.176007	1.174505	22.40013333333333
Dev.			1.192883E-03	5.690592E-04	
Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
U 232	2	0.047572	0.0484	4.70E-02	1
	4	0.15657	0.157657	0.155258	3.257376033057851
	6	0.3226	0.323161	0.32168	6.676880165289256
	8	0.541	0.540416	0.541253	11.16561983471074
	10	0.8058	0.804898	0.80718	16.63012396694215
	12	1.1115	1.112082	1.110849	22.97690082644628
Dev.			1.600131E-03	3.079704E-04	
Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
U 234	2	0.043498	0.0443	4.31E-02	1
	4	0.143351	0.144488	0.1424	3.261580135440181
	6	0.296071	0.296515	0.295231	6.69334085778781
	8	0.49704	0.496465	0.497205	11.2068848758465
	10	0.7412	0.740389	0.742386	16.71306997742664
	12	1.0238	1.02434	1.023252	23.12279909706546
			1.570616E-03	2.365677E-04	
Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
U 236	2	0.045242	0.0460	0.044946	1
	4	0.149476	0.150615	0.148818	3.274239130434783
	6	0.309784	0.310307	0.309214	6.745804347826087
	8	0.52224	0.521697	0.522359	11.34123913043478
	10	0.7823	0.781365	0.783104	16.98619565217391
	12	1.0853	1.085892	1.084928	23.60634782608696
Dev.			1.573864E-03	1.623419E-04	

Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
U 238	2	0.044916	0.0457	4.46E-02	1
	4	0.14838	0.149382	0.1476	3.268752735229759
	6	0.30718	0.307782	0.306694	6.734835886214442
	8	0.5181	0.51748	0.518127	11.32341356673961
	10	0.7759	0.775092	0.776808	16.96043763676149
	12	1.0767	1.077237	1.076291	23.57192560175055
Dev.			1.45654E-03	1.7979E-04	
Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Pu 236	2	0.04463	0.0453	4.43E-02	1
	4	0.14745	0.148484	0.146797	3.277792494481236
	6	0.3058	0.306207	0.305179	6.75953642384106
	8	0.5157	0.515312	0.515928	11.3755408388521
	10	0.7735	0.772594	0.774221	17.05505518763797
	12	1.0743	1.074848	1.073949	23.7273289183223
Dev.			1.411047E-03	1.642599E-04	
Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Pu 238	2	0.044076	0.0446	4.38E-02	1
	4	0.145952	0.146621	0.145334	3.287466367713004
	6	0.30338	0.303648	0.302856	6.808251121076233
	8	0.51358	0.513246	0.5137	11.50775784753363
	10	0.77348	0.772983	0.774211	17.33145739910314
	12	1.0801	1.080427	1.07976	24.22482062780269

Dev.			9.504557E-04	1.490073E-04	
Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Pu 240	2	0.042824	0.0434	4.26E-02	1
	4	0.14169	0.142548	0.141136	3.284516129032258
	6	0.294319	0.294661	0.293798	6.789423963133641
	8	0.49752	0.497096	0.497607	11.45382488479263
	10	0.7478	0.747173	0.748532	17.21596774193548
	12	1.0418	1.042212	1.041465	24.01410138248848
Dev.			1.171425E-03	1.399778E-04	
Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Pu 242	2	0.04454	0.0452	4.43E-02	1
	4	0.1473	0.148435	0.146966	3.28396017699115
	6	0.3064	0.306807	0.305919	6.787765486725664
	8	0.5181	0.517545	0.518096	11.45011061946903
	10	0.7786	0.777846	0.779275	17.20898230088496
	12	1.0844	1.084906	1.084106	24.00234513274336
Dev.			1.429733E-03	1.069978E-04	
Nucleus	I^+	E_{exp}	E_{NS2}	E_{AB}	$R=E_{NS}(I)/E_{NS}(2)$
Pu 244	2	0.0442	0.0471	0.046002	1
	4	0.155	0.154156	0.152371	3.272951167728238
	6	0.3179	0.317914	0.316786	6.749766454352442
	8	0.535	0.535028	0.535599	11.35940552016985
	10	0.8024	0.802177	0.803831	17.03135881104034
	12	1.1159	1.116038	1.115182	23.69507430997877
Dev.			1.99043E-03	1.048036E-04	

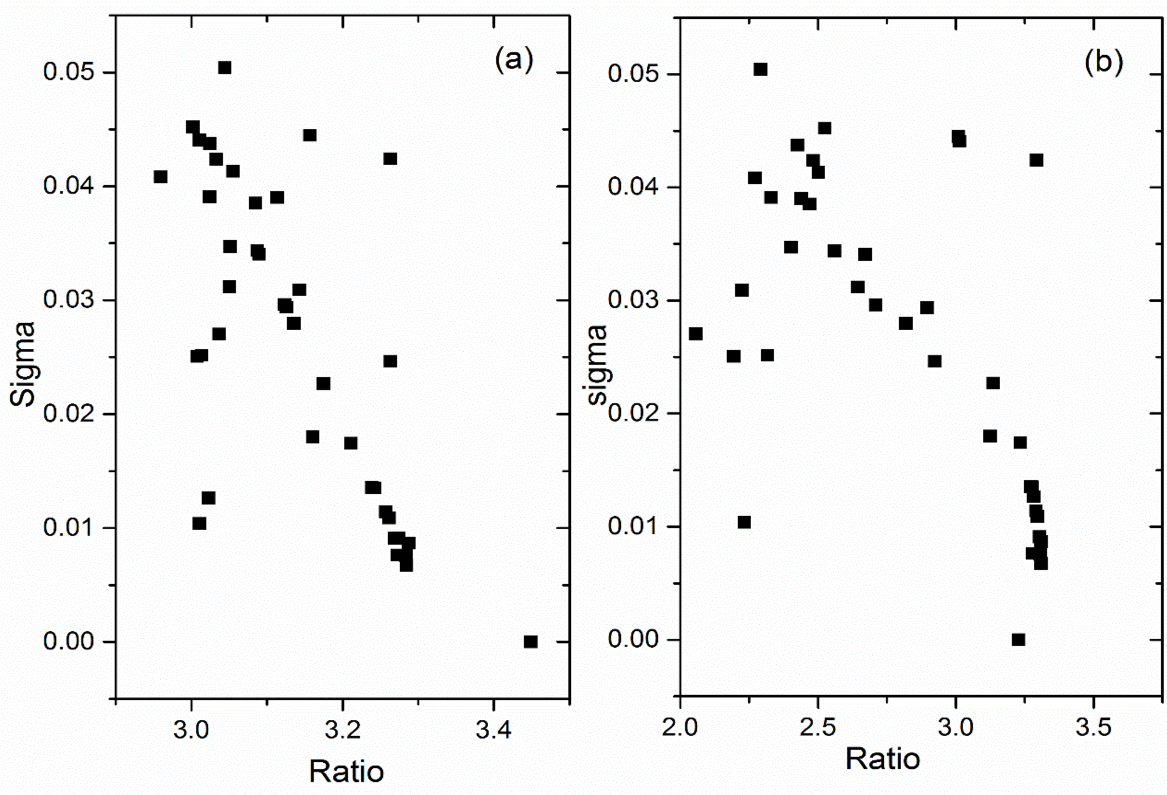

Fig. Sigma (σ_1) vs. Energy ratio (R4)

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