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Dialectical Logic K-Model: Some Applications by Fuzzy-Probability Theory, Cause -Effect Analysis, Chaos Dynamics and Optimization Theory

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Abstract

The paper is a further successor about some problems in dialectical logic K-model which maybe a new useful tool in Artificial Intelligence. The first problem is for the case that true value function is produced by the method of fuzzy-probability theory, this discussion is based on a famous work by Zadeh L. A. (Zadeh L. A., 1965); the second problem is a result about cause-effect analysis, a proof is given for the theorem "a regular cause-effect graph must be a tree-graph", therefor any mathematical formula can be expressed into a cause-effect tree-graph; the third problem is research for chaos dynamics of contradiction function onto magnitude-frequency characteristics; the last problem is discussion about optimization in dialectical logic K-model and a mathematical model about optimization is built up.

Keywords: Dialectical logic K-model; Fuzzy-probability; Cause-effect analysis; Chaotic condition; Optimization model

1. Introduction

There exists a problem in which the power function only can be evaluated via the fuzzy mathematics in dialectical logic K-model which may play a new important role in artificial intelligence(Yaozhi Jiang.,2017)(Yaozhi Jiang.,2018[1])(Yaozhi Jiang.,2018[2]), thus we give some results via fuzzy mathematics in Section 2.. In order to solve the cause-effect relationship, in Section 3. we discuss the cause-effect analysis which is useful in artificial intelligence and can represent any mathematical formula into a cause-effect graph and vise verse. In the Section 4. we discuss a chaotic dynamical result about 2-order dynamical magnitude-frequency characteristics equation of contradiction function and discuss how to judge whether any 2- order dynamical system would be chaotic or not. In the Section 5. we give a optimization model about dialectical logic K-model.

2. Fuzzy-probability theory applied to dialectical logic K-model

In dialectical logic K-model, it is possible that the power function is only evaluated by fuzzy mathematics(Zadeh L. A., 1965), thus we must expand the fuzzy theory by some new concepts, such as multidimensional membership functions, stochastic process membership function, multidimensional stochastic process membership functions acted on Euclidean space, etc. In this section we call the joint name of the fuzzy theory reformed by probability as fuzzy-probability. In the fuzzy-probability theory: either events or their probability are fuzzy.

For set $A \subseteq \Psi$, whether set A is countable or not, in which Ψ is subjective domain,

$\forall a_i \in A$, $\exists F^t$, F^t is a stochastic process mapping, makes

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$F^t : A \subseteq \Psi \to [0,1]$, and $F^t(a_i \in A) \mapsto [0,1]$

Then we call the $F^t(a_i)$ is the stochastic process membership function of element a_i, or

$F^t(a_i)$ is the stochastic process membership degree of element a_i to the set A.

Some stochastic process membership function is called multidimensional, if and only if its stochastic process membership function is many-factorized.

We denote the q-dimensional membership function of set $A = \{a_1, a_2, \cdots, a_m\}$ by

$F_q^t : A = \{F_1^t : A, F_2^t : A, \cdots, F_q^t : A\} \to [0,1]$

q-dimensional membership function is a q-element mapping index set.
If the multidimensional membership function is acted on 3-dimensional Euclidean space A^3, then we denote it by

$F_q^t : A^3 = \{F_1^t : A^3, F_2^t : A^3, \cdots, F_q^t : A^3\} \to [0,1]$

Especially, if $F_q^t = \{F_1^t, F_2^t, \cdots, F_q^t\}$, and fuzzy-probability membership function set $P^F = \{P_1^F, P_2^F, \cdots, P_q^F\}$, $\forall F_s^t$, $s = 1,2,\cdots,q$, always exist one-to-one mapping Φ, make $\Phi : F_q^t \to P_q^F$.

Fuzzy-probability membership function is produced by probability distribution function of time-varying membership function.

Definition 2.1. For a countable and finite set $A \subseteq \Psi$, and a probability density function set $P^F = \{P_1^F, P_2^F, \cdots, P_q^F\}$ (where we define directly the fuzzy-probability membership function by probability density function), if exists a q-dimensional stochastic process membership mapping F_q^t and a one-to-one mapping Φ, make

$$\begin{aligned}
&\Phi : (F_q^t : A) = \Phi : (\{F_1^T : A, F_2^t : A, \cdots, F_q^t : A\} \to [0,1]) = \\
&= \{\Phi_1 : (F_1^t : A), \Phi_2 : (F_2^t : A), \cdots, \Phi_q : (F_q^t : A)\} \to [0,1] = \\
&= \{P_1^F : A, P_2^F : A, \cdots, P_q^F : A\} \to [0,1] = \\
&= \{P_1^F : \{a_i\}, P_2^F : \{a_i\}, \cdots, P_q^F : \{a_i\}\} \to [0,1]
\end{aligned}$$

In the formula above, $i = 1,2,\cdots,m$, we call that is a m-element q-dimensional fuzzy-probability. If exists dividing mapping Θ, $\forall A \subseteq \Psi$, make

$\Theta : A = \{A_1, A_2, \cdots, A_r\} \to [0,1]$ and $\forall q$, $\exists$ $\sum_{j=1}^{r} F_{q,j}^t(A_j) = 1$ then we call

$\Phi : (F_q^t : (\Theta : A)) \to [0,1]$

is a m -element r -phase q -dimensional fuzzy-probability set, and denote it by $A(m,r,q,)$.

Definition 2.2. The Boolean operators $\bigcup , \bigcap$ and N

If $A(m,i,q), A(m,j,q) \subseteq \Psi$, and $i, j \in r$, then

A. $A(m,i,q) \bigcup A_j(m,j,q) = \max\{F_i^t : A(m,i,q), F_j^t : A(m,j,q)\}$,

or $\bigcup_{i=1}^{r} A(m,i,q) = \max\{F_i^t : A(m,i,q)\}$,

B. $A(m,i,q) \bigcap A(m,j,q) = \min\{F_i^t : A(m,i,q), F_j^t : A(m,j,q)\}$,

or $\bigcap_{i=1}^{r} A(m,i,q) = \min\{F_i^t : A(m,i,q)\}$,

C. If $\bigcup_{i=1}^{r} A_i = A$, then

$N(F_i^t : A(m,i,q)) = 1 - (F_i^t : A(m,i,q))$,

Definition 2.3. In a smooth stochastic time-varying process $y = f(t)$, if derived number $\frac{dy}{dt}$ exist and where $\frac{dy}{dt} = 0$ must exist mini-max value set $M = \{y_1, y_2, \cdots, y_s\}$, and $y_{\max} = \sup M$, $y_{\min} = \inf M$.

We define a crossing function $\kappa(y_{a_1,a_2}, 0 \le t \le t_1)$ is crossed number of $y = f(t)$ by line $y = a_1$ and line $y = a_2$, where a_1 and a_2 all are constant and $a_1 \ne a_2$, between the two line is called as a Poincare sectional planed-box, and its thickness is $\Delta T = a_2 - a_1$ and its length is $\Delta L = t_1$. In a same Poincare sectional planed-box the crossed number on any Poincare sectional plane is a same constant as other Poincare sectional plane, if the line $y = a_1$ or $y = a_2$ just a line of tangent at some mini-max values we must count the crossed number at these tangent points by twice. Obviously $y_{\min} < a_1 < a_2 < y_{\max}$, so we can divide the closed interval $[y_{\min}, y_{\max}]$ into several Poincare sectional planed-boxes $B = \{y_{\min}, a_1; a_1, a_2; a_2, a_3; \cdots; a_{s-1}, a_s; a_s, y_{\max}\}$. We call that the stochastic probability-density function of fuzzy-probability membership function $y_{\min} \le f_i^t(a_{s,s+1} \in A_q) \le y_{\max}$ by $\kappa(y_{a_s,a_{s+1}}, 0 \le t \le t_1) = p(f_i^t(a_{s,s+1} \in A_q))$, then

Property 2.1. $\forall \quad a \in A(m,r,q)$, we have a probability-distribution formula below

a.

$$P(a, 0 \le t \le t_1) = \sum_{y_{\min}}^{y_{\max}} \int_{\Delta y_s} p(f_i^t(a \in A(m,r,q)))dy =$$

$$= \sum_{s=0}^{j} \int_{t_s}^{t_{s+1}} \kappa(y_{a_s,a_{s+1}}, 0 \le t \le t_1) f'(t)dt \in [0,1], j = 1,2,\cdots,$$

b. $P(a \in A(m,r,q), t_1 \le t < \infty) = 1 - P(a \in A(m,r,q), t \le t_1)$,

Property 2. 2. (The definition for fuzzy-probability membership function)
If denote the mathematical expectation of fuzzy-probability membership function by

$$E(f_i^t(a \in A(m,r,q))) \approx \sum_{s=0}^{j} \int_{t_s}^{t_{s+1}} f_i^t(a \in A(m,r,q), 0 \le t \le t_1) \times \kappa(y_{a_s,a_{s+1}}, 0 \le t \le t_1) f'(t)dt$$

a.fuzzy-probability membership function

$$F_i^t(a \in A(m,r,q)) = f_i^t(a \in A(m,r,q)) \times \kappa(y_{a_s,a_{s+1}}, 0 \le t \le t_1) f'(t)$$

i.e.

$$F_i^t(a \in A(m,r,q)) = \frac{d}{dt} E(f_i^t(a \in A(m,r,q)))$$

b. $\lim P(a_{m,i}, t \to \infty) = 1$

c. $\lim(1 - P(a_{m,i}, t \to \infty)) = 0$

Definition 2.4. $\lambda -$ truncated set

If exists a real-number $\lambda \in (0,1)$, make $A_\lambda = \{a_i\}$ for all $F^t(a_i) \le \lambda$, $i \in m$, we call set

A_λ is a $\lambda -$ truncated set. The real-number λ also can be called as threshold value.

Property 2.3. If $\lambda_1 \le \lambda_2$, then $A_{\lambda_1} \subseteq A_{\lambda_2}$.

Definition 2.5. Whole fuzzy degree index of fuzzy-probability set A_i

Denote the q th whole fuzzy degree index I^q of fuzzy-probability set A_i by

$$I^q(A_i) = \left| \sum_{i=0}^{m} \left| f_i^t(a \in A_i) - Ef_i^t(a \in A_i) \right|^2 \right|^{\frac{1}{2}}$$

Now we have done the work that expand the fuzzy-probability method to apply it to the dialectical logic K-model. This method is suitable for the case that the power function is evaluated by fuzzy mathematics method.

3.Cause-effect analysis applied to dialectical logic K-model

Denote the objective-domain by B and the subjective-domain by A separately, then there is a couple of research operator ∇ and inverse operator ∇^{-1} (Yaozhi Jiang., 2017), make

$$(A_1 \in A) \Leftarrow \nabla(B_\alpha \in B)$$

$$\nabla^{-1}(A_1 \in A) = (A_2 \in A) \Rightarrow \nabla^{-1}\nabla(B_\alpha \in B)$$

$$(A_3 \in A) \Leftarrow \nabla\nabla^{-1}\nabla(B_\alpha \in B)$$

$$\nabla^{-1}(A_3 \in A) = (A_4 \in A) \Rightarrow \nabla^{-1}\nabla\nabla^{-1}\nabla(B_\alpha \in B)$$

$$\cdots$$

$$(A_{k+1} \in A) \Leftarrow \nabla\underbrace{\nabla^{-1}\nabla\cdots\nabla^{-1}\nabla}_{k-couples-of-\nabla^{-1}\nabla}(B_\alpha \in B)$$

$$\nabla^{-1}(A_{k+1} \in A) = (A_{k+2} \in a) \Rightarrow \underbrace{\nabla^{-1}\nabla\cdots\nabla^{-1}\nabla}_{(k+1)-couples-of-\nabla^{-1}\nabla}(B_\alpha \in B)$$

$$\cdots$$

In recurrence formulas above, $k = 0,1,2,\cdots,n$ and subjective logic $(A_{k+2} \in A)$ is the mirror image operated by the $(k+1)$ operator couple $\nabla^{-1}\nabla$ from objective logic $(B_\alpha \in B)$, and vice verse.

If denote the k operator couple $\nabla^{-1}\nabla$ by $\Re_k$, i.e.

$$\Re_k = \underbrace{\nabla^{-1}\nabla\cdots\nabla\nabla^{-1}}_{k-couples-of-\nabla^{-1}\nabla}$$

Then the recurrence formulas above is changed into below

$$(A_1 \in A) \Leftarrow \nabla(B_\alpha \in B)$$

$$\cdots$$

$$(A_{k+1} \in A) \Leftarrow \nabla\Re_k(B_\alpha \in B)$$

$$\nabla^{-1}(A_{k+1} \in A) = (A_{k+2} \in a) \Rightarrow \Re_{k+1}(B_\alpha \in B)$$

$$\cdots$$

The research couple operator $\Re_k$ is called k-order circular research operator.

3.1. Expressions in matrix for research operator ∇ and inverse operator ∇^{-1}

We denote the research operator by vector expression $\nabla=(\nabla_1,\nabla_2,\nabla_3,\nabla_4,\nabla_5,\nabla_6)$, in which the sub-operator ∇_1 is self-thinking in the machine, sub-operator ∇_2 is the knowledge communicated inter-machine, sub-operator ∇_3 is the instruments operated by machines, sub-operator ∇_4 is methods used by machines to research, sub-operator ∇_5 is the researching-involved factors in environment , ∇_6 is data known or unknown stored in database of knowledge previously.

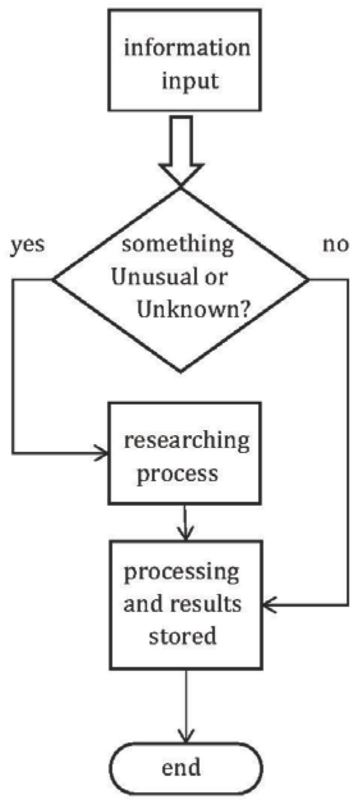

Fig. 3.1. The structure of flow-process diagram of operator $\nabla_i\,(i=1,2,\cdots,6)$

Every one of these six sub-operators are gone through the programming shown in the Fig. 3.1.

In the vector expression of inverse operator $\nabla^{-1}=(\nabla_1^{-1},\nabla_2^{-1},\nabla_3^{-1},\nabla_4^{-1},\nabla_5^{-1},\nabla_6^{-1})$, there also are six sub-operators: ∇_1^{-1} is a sub-operator to check the self-thinking of machine whether true or false back to the objective domain; ∇_2^{-1} is a sub-operator to check the knowledge communicated inter-machine whether true or false back to the objective domain; ∇_3^{-1} is a sub-operator to check the instruments operated by machine whether good or bad back to the objective domain; ∇_4^{-1} is a sub-operator to check the methods used by machine whether good or bad back to objective domain; ∇_5^{-1} is a sub-operator to check the researching-involved factors in environment whether true or false back to objective domain; ∇_6^{-1} is a sub-operator to check the data known or unknown stored in database of knowledge previously whether true or false back to objective domain.

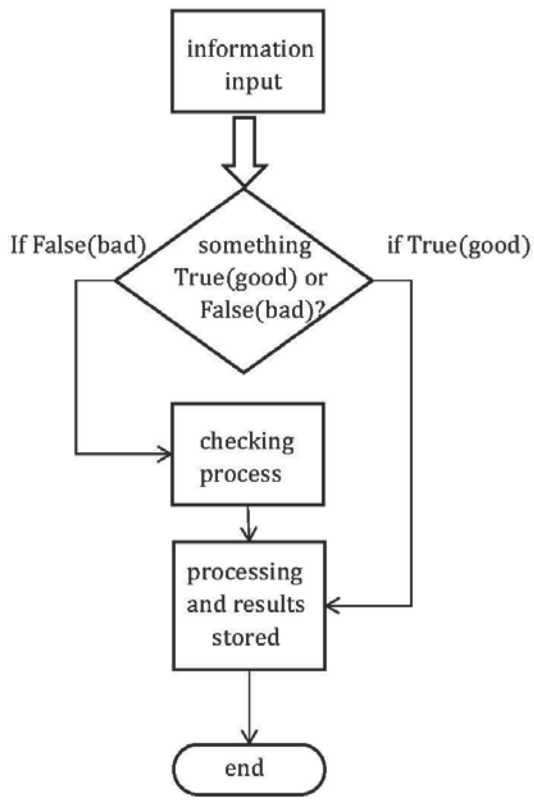

Fig.3.2. The structure of flow-process diagram of inverse operator ∇_i^{-1} ($i = 1,2,\cdots,6$)

3.2. The matrices of research operator ∇ and inverse operator ∇^{-1} in n-order

Denote the research operator ∇ and inverse operator ∇^{-1} in n-order separately by

$$\nabla_n = \begin{vmatrix} \nabla_{1,1} & \nabla_{1,2} & \cdots & \nabla_{1,n} \\ \nabla_{2,1} & \nabla_{2,2} & \cdots & \nabla_{2,n} \\ \nabla_{3,1} & \nabla_{3,2} & \cdots & \nabla_{3,n} \\ \nabla_{4,1} & \nabla_{4,2} & \cdots & \nabla_{4,n} \\ \nabla_{5,1} & \nabla_{5,2} & \cdots & \nabla_{5,n} \\ \nabla_{6,1} & \nabla_{6,2} & \cdots & \nabla_{6,n} \end{vmatrix} \qquad \nabla_n^{-1} = \begin{vmatrix} \nabla_{1,1}^{-1} & \nabla_{1,2}^{-1} & \cdots & \nabla_{1,n}^{-1} \\ \nabla_{2,1}^{-1} & \nabla_{2,2}^{-1} & \cdots & \nabla_{2,n}^{-1} \\ \nabla_{3,1}^{-1} & \nabla_{3,2}^{-1} & \cdots & \nabla_{3,n}^{-1} \\ \nabla_{4,1}^{-1} & \nabla_{4,2}^{-1} & \cdots & \nabla_{4,n}^{-1} \\ \nabla_{5,1}^{-1} & \nabla_{5,2}^{-1} & \cdots & \nabla_{5,n}^{-1} \\ \nabla_{6,1}^{-1} & \nabla_{6,2}^{-1} & \cdots & \nabla_{6,n}^{-1} \end{vmatrix}$$

Then we have

$$\Re_n = \nabla_n^{-1}\nabla_n = \begin{vmatrix} \nabla_{1,1}^{-1}\nabla_{1,1} & \nabla_{1,2}^{-1}\nabla_{1,2} & \cdots & \nabla_{1,n}^{-1}\nabla_{1,n} \\ \nabla_{2,1}^{-1}\nabla_{2,1} & \nabla_{2,2}^{-1}\nabla_{2,2} & \cdots & \nabla_{2,n}^{-1}\nabla_{2,n} \\ \nabla_{3,1}^{-1}\nabla_{3,1} & \nabla_{3,2}^{-1}\nabla_{3,2} & \cdots & \nabla_{3,n}^{-1}\nabla_{3,n} \\ \nabla_{4,1}^{-1}\nabla_{4,1} & \nabla_{4,2}^{-1}\nabla_{4,2} & \cdots & \nabla_{4,n}^{-1}\nabla_{4,n} \\ \nabla_{5,1}^{-1}\nabla_{5,1} & \nabla_{5,2}^{-1}\nabla_{5,2} & \cdots & \nabla_{5,n}^{-1}\nabla_{5,n} \\ \nabla_{6,1}^{-1}\nabla_{6,1} & \nabla_{6,2}^{-1}\nabla_{6,2} & \cdots & \nabla_{6,n}^{-1}\nabla_{6,n} \end{vmatrix}$$

By the matrix $\Re_n$, machine can has abilities to treat a lot of problems.

The superior high-speed convergence property of the two operators is depend on the cleverness of machines conformed the skill stored in associated database(ADB), that is the self-study.

3.3. Cause-effect analysis

Suppose exist a set $X = \{x_1, x_2, \cdots, x_n\}$ and a weighted mapping set $F = \{f_1, f_2, \cdots, f_k\}$ make $F : X \to G$, $f \mapsto (x_i, x_j)$; obviously where $G = \{X; F\}$ is a graph, and in which X is the nodes set and F is the the edges set, then we call G is a cause-effect graph. The Fig 3.3. is an example for cause-effect graph as below:

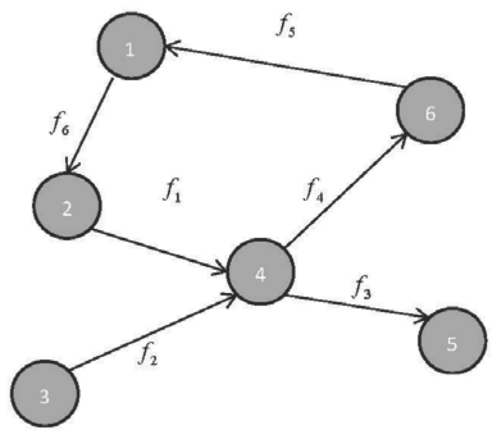

Fig. 3.3. an example of cause-effect graph

In the graph above $i=1,2,\cdots,6$ is denote the nodes : $x_1, x_2, \cdots, x_6$.

3.1. Definition

A. Cause-effect graph is a directed, connected graph without self-loop, and multiple edges are allowable;

B. In any edge the head-node is "cause variable or cause node" and the tail-node is "effect variable or effect node", the node which has only incoming edge is called final-effect node and the node which has only outgoing edge is called initial-cause node. Denote initial-cause node set X_c by $\{x_1, x_2, \cdots, x_s\} = X_c \subset X$, denote the final-effect node set X_e by $\{x_1^*, x_2^*, \cdots, x_q^*\} = X_e \subset X$ and denote the intermediate node set X_i by $\{x_{i,1}, x_{i,2}, \cdots, x_{i,m}\} = X_i \subset X$, of cause $\forall X_c, X_e, X_i$, , all are nonempty set, and $X_c \cap X_e = X_c \cap X_i = X_e \cap X_i = \varnothing$.

C. For any edge, if its cause variable of head-node is x_i and its edge weighted mapping is f_r, then its effect variable of tail-node x_j is $x_j = f_r(x_i)$;

D. Actually, for any edge, if its cause variable of head-node is x_i and its edge weighted mapping is $f_r(x_i)*$, then its effect variable of tail-node x_j is $x_j = f_r(x_i) \times x_i$;

Definition 3.2. For a cause-effect graph $G = \{X, F\}$

A. (Parallel Law) For any node x_j, if the node is n-incoming edges, its head-node cause variable is $x_i, i = 1,2,\cdots,n$, its weighted mapping set is $F_1 = \{f_{1,1}, f_{1,2}, \cdots, f_{1,n}\}$, then

$$x_j = (f_{1,1} \oplus f_{1,2} \oplus \cdots \oplus f_{1,n}) = (f_{1,1} + f_{1,2} + \cdots + f_{1,n})(x_i) = \sum_{p=1}^{n} f_{1,p}(x_i)$$

B. (Serial Law) For the path $P = \{x_r; f_{r-1}\}$ in cause-effect graph, if its head-node is x_i and its tail-node is x_j, then the effect variable $x_j = f_{j-1}(f_{j-2}(f_{j-3} \cdots f_i(x_i))) = \prod_i^j f_s(x_s)$

C. For the operator plus "$\oplus$" and operator multiple "$\otimes$", if $\forall f_i, f_j \in F$, we have $f_i \oplus f_j = (f_i + f_j) \in F$ and $f_i \otimes f_j \in F$, in which "+" is an arithmetic sum.

D. Operator laws for operator"$\oplus$" and "$\otimes$"

a. Operator plus "$\oplus$"commutative law $f_i \oplus f_j = f_j \oplus f_i$

b. Operator plus "$\oplus$" associative law $f_i \oplus (f_j \oplus f_k) = (f_i \oplus f_j) \oplus f_k$

c. Operator multiple"$\otimes$" non-commutation law $f_i \otimes f_j \neq f_j \otimes f_i$

d. Operator multiple "$\otimes$" non-associative law $f_i \otimes (f_j \otimes f_k) \neq (f_i \otimes f_j) \otimes f_k$

e. Hybrid operator distributive law $f_i \otimes (f_j \oplus f_k) = (f_i \otimes f_j) \oplus (f_i \otimes f_k)$

In order to simplify the formula we denote $f_i \oplus f_j$ by $f_i + f_j$ and denote $f_i \otimes f_j$ by $f_i f_j$.

E. $\forall f_i$, if $\exists f_i \otimes f_i^{-1} = 1$, then call the f_i^{-1} is inverse-mapping of mapping f_i, obviously f_i and f_i^{-1} are reciprocal-mapping.

F. $\forall f_i \in F$, if $\exists f_i^{-1} \in F$, then we call the mapping f_i is a reversible mapping, then its correspondent graph is called reversible graph; otherwise its correspondent graph is called irreversible graph.

G. For reversible graph, weighted mapping f_i on some edge is become into its inverse-mapping f_i^{-1} if the correspondent edge is changed its direction.

Theorem 3.1. If exist a circle $C = \{x_i; f_i\}$ in the irreversible cause-effect graph, then

$x_r = f_1 f_2 \cdots f_r(x_r) = \prod_{s=1}^{r} f_s(x_r) \Rightarrow \prod_{s=1}^{r} f_s = 1$, $\Rightarrow$ means "lead to".

Proof:

Case 1. if C is a pure-circle, the pure-circle which is a circle without any branch-path in conjunction with the circle, $\forall \{x_i, x_j\} \in C$, then exist two paths between x_i and x_j, one is path $P(x_i \to x_j)$ and another is path $P(x_j \to x_i)$. According to serial law, if $F(x_i \to x_j) = \prod f_{i \to j} = f^{+1}$, $F(x_j \to x_i) = \prod f_{j \to i} = f^{-1}$, at this time the pure-circle is simplified into a two-circle, i.e. the pure-circle is become into the circle only two nodes and two edges but the direction of the two edges are reciprocal, we have the $x_i = f^{-1}(x_j), x_j = f^{+1}(x_i)$, then $x_i = f^{+1} f^{-1}(x_i)$, of cause $f^{+1} f^{-1} = 1$;

Case 2. if any branch-path exist, suppose it is a incoming branch-path into the node $x_i \in C$ and its mapping $f_s(x_s)$, $x_s \notin C$, will make $x_i = f_s(x_s) + f_j(x_j)$, in which $f_p(x_{i-1}) \in C$ is another incoming edge included in circle C. Follow the Case 3. we have a two-circle: one edge of the two-circle is $F(x_i \to x_j) = \prod f_{i \to j} = f_p \Rightarrow x_j = f_p(x_i)$ and another edge of the two-circle is $F(x_j \to x_i) \Rightarrow x_i = f_s(x_s) + f_j(x_j)$ thus we have

$$\begin{cases} x_j = f_p(x_i) \\ x_i = f_s(x_s) + f_j(x_j) \end{cases}$$

From formula above, we can obtain

$$x_j = f_p\left[f_s(x_s) + f_j(x_j)\right] = f_p f_s(x_s) + f_p f_j(x_j)$$

Therefor we have

$$\begin{cases} f_p f_j = 1 \Rightarrow \begin{cases} f_p = f_j^{-1} \\ f_j = f_p^{-1} \end{cases} \\ f_p f_s(x_s) = 0 \Rightarrow x_s = 0 \end{cases}$$

With another word, if exist some incoming branch-path in a circle, this incoming branch-path will be deleted so that the circle is become a pure-circle, this is just Case 1..

Case 4. If some node in a circle exist a outgoing branch-path at node x_i, this case do not affect the operator $\otimes$, if use method in Case 1. we can obtain a sub-graph: $x_p \Leftrightarrow x_i \to x_s, \{x_p, x_i\} \in C, x_s \notin C$, the symbol "$\Leftrightarrow$" denotes the 2-circle (x_p, x_i) and the symbol "$\to$" denotes the edge (x_i, x_s), as same as Case 1. we can delete the 2-circle because mappings on 2-circle are reciprocal. Therefor we obtain $f_{p\to i} f_{i\to p} = 1$ and a edge (x_i, x_s).

Proof is over.

Theorem 3.2. For a reversible cause-effect graph, if exist any inverse direction edges of multiple edge between node x_i and x_j, we must changed its direction of these inverse edges and make their correspondent weighted mapping f_i become into f_i^{-1}, then whole mapping $F(x_i, x_j) = \sum f_r + \sum f_s^{-1}$.

Proof: the operation is to avoid appearance of the 2-circle, i.e. circle with only two nodes and two edges. The mappings on a 2-circle are led to the product of operator multiple $\otimes$ is 1. To see the Theorem 3.1. . If we change the inverse direction of multiple edges and inverse their mappings f_s into f_s^{-1}, then we change the cause-effect graph into another cause-effect graph, because we change some intermediate nodes into cause-node or effect-node. At this time, we use Parallel Law can obtain the result that we require.

Proof is over.

Corollary 3.1. The irreversible or reversible cause-effect graph all are tree-graph, but multiple edges in same direction are allowable because they can be merged into one edge which mapping is equal to the arithmetic sum of every edge in multiple edges.

Definition 3.3. A cause-effect graph G is called regular graph, if and only if:

the graph G is a connected, directed tree-graph.

Corollary3.2. For a reversible or irreversible regular cause-effect graph, $\forall x_i, x_j^*$, $x_i \in X_c = \{x_1, x_2, \cdots, x_s\}$, $x_i^* \in X_e = \{x_1^*, x_2^*, \cdots, x_q^*\}$, only one path from $x_i, i = 1,2,\cdots,s$ to $x_j^*, j = 1,2,\cdots,q$ exists.

Proof: use refutation proof method, if exists two or more than two paths in a cause-effect graph, then must exist a circle in the cause-effect graph, this contradicts with "the cause-effect graph is a tree-graph".

Proof is over.

Example 3.1. Recurrence formula of cause-effect graph in Fig.3.3.

For the cycle

$$x_4 = f_1(x_2), x_2 = f_6(x_1), x_1 = f_5(x_6), x_6 = f_4(x_4)$$
$$\therefore x_4 = f_1 f_6 f_5 f_4(x_4) \Rightarrow f_1 f_6 f_5 f_4 = 1$$

For the path $x_3 \to x_4 \to x_5$,

$$x_4 = f_2(x_3), x_5 = f_3(x_4)$$
$$\therefore x_5 = f_3 f_2(x_3)$$

If $f_3 = \frac{d}{dt}, f_2 = \ln$, then $x_5 = f_3 f_2(x_3) = f_3(\ln x_3) = \frac{d}{dt}\ln x_3 = \frac{1}{x_3}\frac{dt}{dx_3}$

If $f_3 = e^x$, $f_2 = \sin x$, then $x_5 = f_3 f_2(x_3) = f_3(\sin x_3) = e^{\sin x_3}$

As above, x_3 is the initial-cause node and x_5 is the final-effect node.

Now we have seen that cause-effect graph can be divided into four types: multiple-cause-to-multiple-effect, multiple-cause-to-single-effect, single cause-to multiple-effect and single-cause-to-single-effect.

Under the more general condition we have had a series of sampled or evaluated datum of cause-effect relationship, need seek the cause-effect graph from the data.

Suppose that a series of datum set is $\mathrm{A}_{s,q} = \{\alpha_1, \alpha_2, \cdots, \alpha_n\} \Rightarrow x_q^* = f_{s,q}(x_s) \cong A_{s,q}$, then we can obtain a cause-effect path matrix

$$H = \begin{vmatrix} A_{1,1} & A_{1,2} & \cdots & A_{1,q} \\ A_{2,1} & A_{2,2} & \cdots & A_{2,q} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s,1} & A_{s,2} & \cdots & A_{s,q} \end{vmatrix} = \begin{vmatrix} f_{1,1}(x_{1,1}) & f_{1,2}(x_{1,2}) & \cdots & f_{1,q}(x_{1,q}) \\ f_{2,1}(x_{2,1}) & f_{2,2}(x_{2,2}) & \cdots & f_{2,q}(x_{2,q}) \\ \vdots & \vdots & \vdots & \vdots \\ f_{s,1}(x_{s,1}) & f_{s,2}(x_{s,2}) & \cdots & f_{s,q}(x_{s,q}) \end{vmatrix}$$

In the matrix above row vector is $X_e = (x_1^*, x_2^*, \cdots, x_q^*)$ and column vector is $X_c = (x_1, x_2, \cdots, x_s)$.

We can solve out the mappings $f_{s,q}$ in matrix H via data-analysis.

4. Chaos dynamics applied to dialectical logic K-model

Chaos dynamics is a research tool to solve that topology properties of solution-stability in the system of nonlinear integral-differential equations. In dialectical logic K-model there are a huge of such problems produced by contradiction function. In this section we try to use chaos dynamics to solve the problem.

4.1. Definitions and symbols

Dynamical system of dialectical logic K-model is a non-autonomous nonlinear system. Chaos produced not only by initial value but also by final value. Denote contradiction function acted on some space by $C_i(\Delta)$, then

$$\begin{cases} T_i(\Delta) = I_i(\Delta) C_i(\Delta) \\ \sum_{i=1}^{n} I_i(\Delta) = I \\ C_i(\Delta) = R + \int_0^t \eta(\Delta) d\Delta + j\frac{\omega^2 LC - 1}{\omega C}, j = \sqrt{-1} \end{cases}$$

In which $I_i(\Delta)$ is flow function of i th edge, $T_i(\Delta)$ is true value function on i th edge,

I is whole flow constant and $\Delta = \begin{cases} x, y, z; t \\ s; t \\ t \end{cases}$

$R_{i,1}(\Delta) = R + \int_0^t \eta(\Delta) d\Delta$ is a real-function, ω is exciting circular frequency, $L(\Delta)$ and

$C(\Delta)$ all are natural parameters of dynamical system.

If the logic variable Δ is acted on 3-dimensional Euclidean space, i.e. $\Delta = x, y, z; t$, in this case the dynamical system is called as Δ_1 − type; if logic variable Δ is acted on logical networks space, i.e. $\Delta = s; t$, in this case the dynamical system is called as Δ_2 − type; if logic variable Δ is independent of any geometric space, i.e. $\Delta = t$, in this case the dynamical system is called as Δ_3 − type.

4.1.1. Integral-differential equations and magnitude-frequency characteristics of Δ_1 − type dynamical system

2-order 4-independent-variable integral-differential equations general expression of Δ_1 − type dynamical system

$$L(x, y, z)C(x, y, z)\frac{d^2u}{dt^2} + \left(\iiint_{x,y,z\in\Delta_1} r(x, y, z)dxdydz + \int_0^t \eta(x, y, z; t)dt \right) C(x, y, z)\frac{du}{dt} +$$
$$+ u(x, y, z; t) + c(x, y, z; t) = 0$$

In which

$$\begin{cases} L(x, y, z) = \iiint_{x,y,z\in\Delta_1} l(x, y, z)dxdydz \\ C(x, y, z) = \iiint_{x,y,z\in\Delta_1} c(x, y, z)dxdydz \end{cases}$$

Its contradiction function

$$C_i(x, y, z; t) = \left(\iiint_{x,y,z\in\Delta_1} r(x, y, z)dxdydz + \int_0^t \eta(x, y, z; t)dt \right) + j\frac{\omega^2 L(x, y, z)C(x, y, z) - 1}{\omega C(X, Y, Z)}$$

Initial-final value condition $t \in [0, t_1], \omega \in [0, \omega_r]$.

Boundary condition $x, y, z \in [(0,0,0), (x_1, y_1, z_1)]$

4.1.2. Integral-differential equations and magnitude-frequency characteristics of Δ_2 − type dynamical system

2-order 2-independent-variable integral-differential equations expression of Δ_2 − type dynamical system

$$L_s C_s \frac{d^2 u}{dt^2} + \left(\int_{s_1}^{s_2} r(s;t)ds + \int_0^t \eta(s;t)dt \right) C_s \frac{du}{dt} + u(s;t) + c(s;t) = 0$$

In which

$$\begin{cases} L_s = \int_{s_1}^{s_2} l(s)ds \\ C_s = \int_{s_1}^{s_2} c(s)ds \end{cases}$$

Its contradiction function is

$$C(s;t) = \left(\int_{s_1}^{s_2} r(s;t)ds + \int_0^t \eta(s,t)dt \right) + j\frac{\omega^2 L(s)C(s) - 1}{\omega C(s)}$$

Initial-final value condition $t \in [0, t_1], \omega \in [0, \omega_r]$.

Boundary condition $s \in [s_1, s_2]$

4.1.3. Integral-differential equations and magnitude-frequency characteristics of Δ_3 − type dynamical system

2-order 1-independe-variable integral-differential equations expression of Δ_3 − type dynamical system

$$LC\frac{d^2 u}{d\Delta_3{}^2} + \left(R(\Delta_3) + \int_0^{\Delta_3} \eta(\Delta_3)d\Delta_3 \right) C\frac{du}{d\Delta_3} + u(\Delta_3) + c(\Delta_3) = 0$$

Its contradiction function

$$C_i(\Delta_3) = \left(R(\Delta_3) + \int_0^{\Delta_3} \eta(\Delta_3)d\Delta_3 \right) + j\frac{\omega^2 LC - 1}{\omega C}$$

Initial-final value condition $t \in [0, t_1], \omega \in [0, \omega_r], L > 0, C > 0$.

4.2. Chaos dynamical analysis of contradiction function $C_i(\Delta)$

As show above, contradiction function obviously is a non-linear function.

4.2.1. Regular expression of contradiction function $C_i(\Delta)$

$$\begin{cases} T_i(\omega) = I_i(\omega)C_i(\omega) \\ \sum_{i=1}^{n} I_i(\omega) = I \\ LC\frac{d^2u}{dt^2} + \left(R + \int_0^{t_1}\eta(t)dt\right)C\frac{du}{dt} + u(t) + c(t) = 0 \\ t = f(\omega) \\ C_i(\omega) = \left(R + \int_0^{\omega_1}\eta(\omega)f'(\omega)d\omega\right) + j\frac{\omega^2 L(\omega)C(\omega) - 1}{\omega C(\omega)} \\ t \in [0, t_1], \omega \in [\omega_s, \omega_r] \end{cases}$$

In which, we have replaced the logic variable Δ by exciting circular frequency ω.

4.2.2. Lyapunov Index of the contradiction function $C_i(\omega)$

Theorem 4.1. Denote all of peak values of magnitude-frequency characteristics $A_f(\omega)$ from contradiction function $C_i(\omega)$ by set $A_\omega = \{\omega_1, \omega_2, \cdots, \omega_p\}$, if exciting circular frequency $\omega \in A_\omega$ and its Lyapunov index $I(\omega) > 0$, then where dynamical system is chaotic.

Proof: "peak value" means that at this point the one-order derived number of the magnitude-frequency characteristics $A_f(\omega)$ is

$$A'_f(\omega) = \begin{cases} 0 \\ \infty \\ discontinous \end{cases}$$

According to the definition of Lyapunov index:

$$I(\omega_r) = \lim_{\omega \to \omega_r} \sup \frac{\ln A'_f(\omega)}{\omega}$$

If $I(\omega_r) > 0$, of cause the dynamical system is chaotic, though we can obtained the result.

Proof is over.

4.2.3.The calculation for Lyapunov index of magnitude-frequency characteristics $A_f(\omega)$ from contradiction function $C_i(\omega)$

$$\begin{cases} A_f(\omega)=\sqrt{\left(R+\int_{\omega_0}^{\omega}\frac{\eta(\omega)}{f'(\omega)}d\omega\right)^2+\left(\frac{\omega^2L(\omega)C(\omega)-1}{\omega C(\omega)}\right)^2} \\ \frac{\partial A_f(\omega)}{\partial\omega}=\frac{\sqrt{2}}{4\sqrt{\left(R+\int_{\omega_0}^{\omega}\frac{\eta(\omega)}{f'(\omega)}d\omega\right)\frac{\eta(\omega)}{f'(\omega)}+\frac{\omega^4L^2(\omega)C^2(\omega)-1}{\omega^3C^2(\omega)}}} \\ \frac{\partial A_f(\omega)}{\partial L(\omega)}=\frac{\sqrt{2}}{4\sqrt{\frac{\omega^2L(\omega)C(\omega)-1}{C(\omega)}}} \\ \frac{\partial A_f(\omega)}{\partial C(\omega)}=\frac{\sqrt{2}}{4\sqrt{\left(\frac{\omega^2L(\omega)C(\omega)-1}{\omega^2C^3(\omega)}\right)}} \end{cases}$$

Make $\frac{\partial A_f(\omega)}{\partial\omega}=\begin{cases}0\\\infty\end{cases}$, $\frac{\partial A_f(\omega)}{\partial L(\omega)}=\begin{cases}0\\\infty\end{cases}$, $\frac{\partial A_f(\omega)}{\partial C(\omega)}=\begin{cases}0\\\infty\end{cases}$, then we can obtain a series of peak values and according to these we can calculate the correspondence Lyapunov index.

Remark: It must be mentioned that the Lyapunov index is possible to be a function.

Section 4. is a further researching successor from the "the critical-point theorem" in author's early works(Yaozhi Jiang., 2017).

5.Optimization theory applied to dialectical logic K-model

We have three sorts of problems to solve in this section, i.e. maximizing the true value function under the condition that minimizing its cost function or minimizing its error function, or their hybrid problem. These three sorts of problems above can be called by a joint name: the optimization problem, and can also be called separately: the cost optimization problem, error optimization problem and hybrid optimization problem.

In this section, we can use the Mozi principle(Yaozhi Jiang.,2017), the mini-max principle to treat the three sorts of problems.

However in the three problems whether their true value functions are continuous or discrete, we would use the definitions for Boolean logic operator onto the optimization problem(Yaozhi Jiang, 2017). With another word, the Boolean logic operator can be used either continuous case or discrete case.

5.1. Boolean logic operators

A. The optimization to true value function(application to Boolean OR)

In general games machine hopes the true value function of own side is maximal, i.e.

$$\bigcup_{i=1}^{s}T\left(A_i^{\pm n}(t)\right)=\max\left\{T\left(A_1^{\pm n}(t)\right),T\left(A_2^{\pm n}(t)\right),\cdots,T\left(A_s^{\pm n}(t)\right)\right\}$$

B. The optimization to cost function(application to Boolean AND)

In general games machine hopes the cost function of own side is minimal, i.e.

$$\bigcap_{s=1}^{i}C\left(T\left(A_s^{\pm n}(t)\right)\right)=\min\left\{C\left(T\left(A_1^{\pm n}(t)\right)\right),C\left(T\left(A_2^{\pm n}(t)\right)\right),\cdots,C\left(T\left(A_i^{\pm n}(t)\right)\right)\right\}$$

C. The optimization to error function(application to Boolean AND)

In general games machine hopes the error function of own side is minimal, i.e.

$$\bigcap_{s=1}^{i} E\left(T\left(A_s^{\pm n}(t)\right)\right) = \min\left\{E\left(T\left(A_1^{\pm n}(t)\right)\right), E\left(T\left(A_2^{\pm}(t)\right)\right), \cdots, E\left(T\left(A_i^{\pm n}(t)\right)\right)\right\}$$

5.1. A general mathematical expression for optimization problem

For the first kind of problem, its general expression is shown below:

$$\bigcup_{i=1}^{s} T\left(A_i^{\pm n}(t)\right): \left|\bigcap_{i=1}^{s} C\left(T\left(A_i^{\pm}(t)\right)\right)\right|$$

For the second kind of problem, its general expression is shown below:

$$\bigcup_{i=1}^{s} T\left(A_i^{\pm n}(t)\right): \left|\bigcap_{i=1}^{s} E\left(T\left(A_i^{\pm}(t)\right)\right)\right|$$

For the third kind of hybrid optimization problem, its general expression is shown below:

$$\bigcup_{i=1}^{s} T\left(A_i^{\pm n}(t)\right): \left|\bigcap_{i=1}^{s} C\left(T\left(A_i^{\pm}(t)\right)\right)\right| \oplus \left|\bigcap_{i=1}^{s} E\left(T\left(A_i^{\pm n}(t)\right)\right)\right|$$

5.2. The general principles and analysis onto the optimization model

The Mozi principle has told us(Johnston Ian., 2010)(Yaozhi Jiang., 2017): every varying process always is satisfied by or satisfied asymptotically by that the gain-function is maximum and at same time the cost-function is minimum.

Actually the solutions for the optimization problem often are not a constant but a function, even sometimes a functional analysis. The solutions are time-varying function and produced by operating circular research operator $\Re$ and Boolean logic operators.

Now we can build the optimization analysis model for dialectical logic K-model below.

$$\min f_c(\Delta)$$

$$s.t.\begin{cases} T\left(A_i^{\pm n}(\Delta)\right)\begin{Bmatrix} < \\ = \\ > \end{Bmatrix} T^*\left(A_i^{\pm n}(\Delta)\right) \\ (\Delta_1) = (x, y, z; t) \in (X, Y, Z; T) \\ (\Delta_2) = (s; t) \in (S; T) \\ (\Delta_3) = (t) \in (T) \\ G = \left\{G^+ \Leftrightarrow G^-\right\} \\ O(\Delta) \in \{o(\Delta)\} \end{cases}$$

In which, $\min f_c(\Delta)$ is minimizing its objective function $f_c(\Delta)$; $T^*\left(A_i^{\pm n}(\Delta)\right)$ is the particular value of true value function; (Δ_1), (Δ_2) and (Δ_3) are the boundary-initial value condition; $G = \left\{G^+ \Leftrightarrow G^-\right\}$ denote the game matrix condition between positive side and negative side; $O(\Delta)$ is other subjective condition.

There are many of books about optimization-method, we do not discuss the methods to solve the optimization analysis model above.

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