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RFID Tags Detectors Stability Analysis Under Delayed Schottky Diode's Internal Elements in Time

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***ABSTRACT* — In this article, we discuss the crucial subject of stability analysis of RFID tag detectors under Schottky diodes internal time delay elements. The Schottky diode detector demodulates the signal and sends the data on to the digital circuit of the TAG; this is the so-called "wake up" signal. A simple RFID TAG receiver block diagram includes input antenna signal with series resistance, inductor (choke), Schottky diode, and output capacitor. Due to the Schottky parasitic delay, there is a stability issue in analyzing detector operation. We define τ_1, τ_2 as delays in time respectively for a Schottky equivalent circuit. We first consider those two delays in time that are not equal ($\tau_1 \neq \tau_2$)then the other three cases $\tau_1 = \tau$ and $\tau_2 = 0$, $\tau_2 = \tau$ and $\tau_1 = 0$, $\tau_1 = \tau$ and $\tau_2 = \tau$. The RFID receiver detector time delay equivalent circuit can be represent as delayed differential equations that depend on variable parameters and delays. The article illustrates certain observations, and analyzes local bifurcations of an appropriate arbitrary scalar delayed differential equation. All of that for optimization of an RFID receiver detector equivalent circuit parameters analysis to get the best performance.**

Index Terms: RFID video receiver, Schottky diode, Delay Differential Equations (DDE), Stability, Bifurcation, Orbit.

I. Introduction

In this article, we discuss the crucial and useful subject of stability analysis of RFID tag detectors under Schottky diodes internal time delay elements. In RFID systems, the reader or interrogator sends a modulated RF signal that is received by the TAG. The Schottky diode detector demodulates the signal and sends the data on to the digital circuits of the TAG. The reader stops sending modulated data and illuminates the TAG with continuous wave (CW) or an un-modulated signal. The TAG's FSK encoder and switch driver, switch the load placed on the TAG's antenna from one state to another, causing the radar cross section of the TAG to be changed. For incoming RF small signal from the RFID reader to the TAG, we can use Schottky diode which represented by a linear equivalent circuit. Rj is the junction resistance (Rv or video resistance) of the diode, where RF power is converted into video voltage output. For maximum output, all the incoming RF voltage should ideally appear across Rj. Cj is the junction capacitance

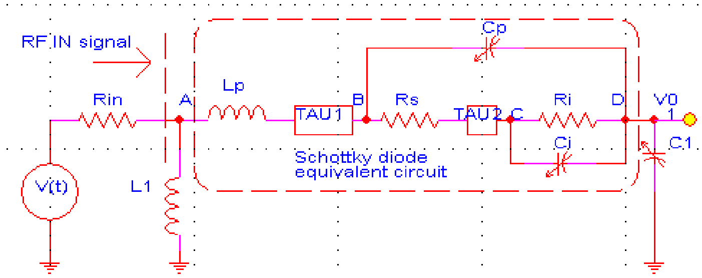

Fig. 1. RFID TAG receiver detector equivalent circuit.

of the diode chip itself. It is a parasitic element which shorts out the junction resistance, shunting RF energy to the series resistance Rs. Rs is a parasitic resistance representing losses in the diode's bond wire, the bulk silicon at the base of the chip and other loss mechanisms. RF voltage appearing across Rs results in power lost as heat. Lp and Cp are package parasitic inductance and capacitance, respectively. The package parasitic inductance Lp has a parasitic delay element in time (τ_1). The resistance losses in the diode's bond wire have a parasitic delay element in time (τ_2). $V(t)$ represents the RFID tag antenna voltage in time, incoming RF small signal from RFID reader [1] [2]. We consider ideal delay lines (TAU1, TAU2).

$$V_{\tau_1} \to \varepsilon_1; V_{\tau_1} \to \varepsilon_2; \varepsilon_1, \varepsilon_2 \ll \varepsilon \tag{1}$$

II. RFID tag receiver detector equivalent circuit differential equations and fixed points

$$\begin{gathered} \frac{V(t)-V_A}{R_{in}} = I_{R_{in}}; I_{R_{in}} = I_{L_1} + I_{L_P}; V_{\tau_1} \to \varepsilon_1 \\ V_{\tau_1} \to \varepsilon_2; \varepsilon_1, \varepsilon_2 \ll \varepsilon > 0; V_A - V_B = L_P \frac{dI_{L_P}}{dt} \end{gathered} \tag{2}$$

$$\begin{gathered} I_{L_P} = I_{C_P} + I_{R_S}; I_{R_S} = \frac{V_B - V_C}{R_S}; V_A = L_1 \frac{dI_{L_1}}{dt} \\ I_{C_P} = C_P \frac{d(V_B - V_D)}{dt}; I_{R_j} = \frac{V_C - V_D}{R_j} \end{gathered} \tag{3}$$

$$\begin{gathered} I_{C_j} = C_j \frac{d(V_C - V_D)}{dt}; I_{R_S} = I_{R_j} + I_{C_j} \\ I_{C_1} = C_1 \frac{dV_D}{dt}; I_{C_1} = I_{C_P} + I_{R_j} + I_{C_j} \end{gathered} \tag{4}$$

$$\begin{gathered} \frac{dV_D}{dt} = \frac{I_{C_1}}{C_1}; I_{C_j} = C_j \frac{d(V_C - V_D)}{dt} = C_j[\frac{dV_C}{dt} - \frac{dV_D}{dt}] \\ I_{C_j} = C_j[\frac{dV_C}{dt} - \frac{dV_D}{dt}] \end{gathered} \tag{5}$$

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$$I_{C_P} = C_P\frac{d(V_B - V_D)}{dt} = C_P[\frac{dV_B}{dt} - \frac{dV_D}{dt}] = C_P[\frac{dV_B}{dt} - \frac{I_{C_1}}{C_1}]$$
$$\frac{V(t) - V_A}{R_{in}} = I_{R_{in}} = I_{L_1} + I_{L_P} \quad (6)$$

$$\frac{V(t)}{R_{in}} - \frac{L_1}{R_{in}}\frac{dI_{L_1}}{dt} = I_{L_1} + I_{L_P}; I_{R_{in}} = I_{L_1} + I_{L_P}$$
$$I_{L_1} = I_{R_{in}} - I_{L_P} = \frac{V(t) - V_A}{R_{in}} - I_{L_P} \quad (7)$$

$$\frac{V(t)}{R_{in}} - \frac{L_1}{R_{in}}\frac{d}{dt}[\frac{V(t) - V_A}{R_{in}} - I_{L_P}] = I_{L_1} + I_{L_P} = I_{R_{in}}$$
$$I_{C_P} = C_P[\frac{dV_B}{dt} - \frac{I_{L_P}}{C_1}] \quad (8)$$

$$I_{R_j} = \frac{V_C - V_D}{R_j}; I_{R_j}R_j = V_C - V_D; I_{C_j} = C_j\frac{d(V_C - V_D)}{dt}$$
$$I_{R_j}R_j = C_j\frac{d(I_{R_j}R_j)}{dt} = C_jR_j\frac{dI_{R_j}}{dt} \quad (9)$$

$$I_{L_P} = I_{C_P} + I_{R_S} \Rightarrow I_{C_P} = I_{L_P} - I_{R_S}$$
$$I_{C_1} = I_{C_P} + I_{R_j} + I_{C_j} = I_{L_P} - I_{R_S} + I_{R_j} + I_{C_j} \quad (10)$$

$$I_{R_S} = I_{R_j} + I_{C_j} \Rightarrow I_{C_1} = I_{L_P} - I_{R_S} + I_{R_j} + I_{C_j}$$
$$I_{C_1} = I_{L_P} - (I_{R_j} + I_{C_j}) + I_{R_j} + I_{C_j} = I_{L_P} \quad (11)$$

$$I_{R_{in}} = \frac{V(t) - V_A}{R_{in}} = \frac{V(t)}{R_{in}} - \frac{1}{R_{in}}L_1\frac{dI_{L_1}}{dt}$$
$$I_{R_{in}} = \frac{1}{R_{in}}[V(t) - L_1\frac{dI_{L_1}}{dt}] \quad (12)$$
$$I_{L_1} = I_{R_{in}} - I_{L_P}; I_{C_P} = I_{L_P} - I_{R_S}$$

$$I_{C_1} = I_{L_P}; I_{R_S} = I_{R_j} + I_{C_j}; I_{C_j} = C_j[\frac{dV_C}{dt} - \frac{I_{C_1}}{C_1}]$$
$$I_{C_j} = C_j\frac{d}{dt}[I_{R_j}R_j] = C_jR_j\frac{dI_{R_j}}{dt} \quad (13)$$

$$I_{C_P} = C_P[\frac{dV_B}{dt} - \frac{I_{C_1}}{C_1}]; V_A - V_B = L_P\frac{dI_{L_P}}{dt}$$
$$L_1\frac{dI_{L_1}}{dt} - V_B = L_P\frac{dI_{L_P}}{dt}; V_B = L_1\frac{dI_{L_1}}{dt} - L_P\frac{dI_{L_P}}{dt} \quad (14)$$

$$\frac{dV_B}{dt} = L_1\frac{d^2I_{L_1}}{dt^2} - L_P\frac{d^2I_{L_P}}{dt^2}; I_{C_P} = C_P[\frac{dV_B}{dt} - \frac{I_{C_1}}{C_1}]$$
$$I_{C_P} = C_P[L_1\frac{d^2I_{L_1}}{dt^2} - L_P\frac{d^2I_{L_P}}{dt^2} - \frac{I_{C_1}}{C_1}] \quad (15)$$

$$I_{L_1} = I_{R_{in}} - I_{L_P} = \frac{V(t) - V_A}{R_{in}} - I_{L_P}$$
$$I_{L_1} = \frac{V(t)}{R_{in}} - \frac{L_1}{R_{in}}\frac{dI_{L_1}}{dt} - I_{L_P} \quad (16)$$
$$I_{C_P} = I_{L_P} - I_{R_S}; I_{C_1} = I_{L_P}$$

$$I_{L_1} = \frac{V(t)}{R_{in}} - \frac{L_1}{R_{in}}\frac{dI_{L_1}}{dt} - I_{L_P}$$
$$\frac{dI_{L_1}}{dt} = \frac{1}{R_{in}}\frac{dV(t)}{dt} - \frac{L_1}{R_{in}}\frac{d^2I_{L_1}}{dt^2} - \frac{dI_{L_P}}{dt} \quad (17)$$

$$\frac{L_1}{R_{in}}\frac{d^2I_{L_1}}{dt^2} = \frac{1}{R_{in}}\frac{dV(t)}{dt} - \frac{dI_{L_P}}{dt} - \frac{dI_{L_1}}{dt}$$
$$\frac{d^2I_{L_1}}{dt^2} = \frac{1}{L_1}\frac{dV(t)}{dt} - \frac{R_{in}}{L_1}\frac{dI_{L_P}}{dt} - \frac{R_{in}}{L_1}\frac{dI_{L_1}}{dt} \quad (18)$$

$$I_{R_S} = I_{R_j} + I_{C_j}; \frac{V_B - V_C}{R_S} = I_{R_j} + I_{C_j}$$
$$I_{C_j} = I_{R_S} - I_{R_j}; I_{C_j} = C_jR_j\frac{dI_{R_j}}{dt} \quad (19)$$
$$I_{R_S} - I_{R_j} = C_jR_j\frac{dI_{R_j}}{dt}$$

$$I_{C_P} = C_P[\frac{dV(t)}{dt} - R_{in}\frac{dI_{L_P}}{dt}$$
$$-R_{in}\frac{dI_{L_1}}{dt} - L_P\frac{d^2I_{L_P}}{dt^2} - \frac{I_{C_1}}{C_1}] \quad (20)$$

$$I_{L_P} - I_{R_S} = C_P[\frac{dV(t)}{dt} - R_{in}\frac{dI_{L_P}}{dt} - R_{in}\frac{dI_{L_1}}{dt}$$
$$-L_P\frac{d^2I_{L_P}}{dt^2} - \frac{I_{C_1}}{C_1}] \quad (21)$$

$$I_{C_1} = I_{L_P}; I_{L_P} - I_{R_S} = C_P[\frac{dV(t)}{dt} - R_{in}\frac{dI_{L_P}}{dt}$$
$$-R_{in}\frac{dI_{L_1}}{dt} - L_P\frac{d^2I_{L_P}}{dt^2} - \frac{I_{L_P}}{C_1}] \quad (22)$$

$$I_{R_S} = \frac{V_B - V_C}{R_S}; V_B - V_C = I_{R_S}R_S$$
$$I_{C_P} = C_P\frac{d(V_B - V_D)}{dt}; \frac{I_{C_P}}{C_P} = \frac{d}{dt}(V_B - V_D) \quad (23)$$
$$V_B - V_D = \frac{1}{C_P}\int I_{C_P}dt$$

$$I_{C_j} = C_j\frac{d(V_C - V_D)}{dt} \Rightarrow \frac{I_{C_j}}{C_j} = \frac{d(V_C - V_D)}{dt}$$
$$V_C - V_D = \frac{1}{C_j}\int I_{C_j}dt \quad (24)$$

$$(*)\, V_B - V_D = \frac{1}{C_P}\int I_{C_P}dt \quad (25)$$

$$(**)\, V_C - V_D = \frac{1}{C_j}\int I_{C_j}dt \quad (26)$$

$$(*) - (**) \quad V_B - V_C = \frac{1}{C_P}\int I_{C_P}dt - \frac{1}{C_j}\int I_{C_j}dt$$
$$I_{R_S}R_S = \frac{1}{C_P}\int I_{C_P}dt - \frac{1}{C_j}\int I_{C_j}dt \quad (27)$$

$$I_{R_S}R_S = \frac{1}{C_P}\int I_{C_P}dt - \frac{1}{C_j}\int I_{C_j}dt$$
$$R_S\frac{dI_{R_S}}{dt} = \frac{1}{C_P}I_{C_P} - \frac{1}{C_j}I_{C_j} \quad (28)$$

$$R_S\frac{dI_{R_S}}{dt} = \frac{1}{C_P}I_{C_P} - \frac{1}{C_j}I_{C_j}$$
$$R_S\frac{dI_{R_S}}{dt} = \frac{1}{C_P}(I_{L_P} - I_{R_S}) - \frac{1}{C_j}(I_{R_S} - I_{R_j}) \quad (29)$$

$$R_S\frac{dI_{R_S}}{dt} = \frac{1}{C_P}(I_{L_P} - I_{R_S}) - \frac{1}{C_j}(I_{R_S} - I_{R_j})$$
$$R_S\frac{dI_{R_S}}{dt} = \frac{1}{C_P}I_{L_P} + \frac{1}{C_j}I_{R_j} - I_{R_S}(\frac{1}{C_P} + \frac{1}{C_j}) \quad (30)$$

$$R_S\frac{dI_{R_S}}{dt} = \frac{1}{C_P}I_{L_P} + \frac{1}{C_j}I_{R_j} - I_{R_S}(\frac{1}{C_P} + \frac{1}{C_j})$$
$$\frac{dI_{R_S}}{dt} = \frac{1}{R_SC_P}I_{L_P} + \frac{1}{R_SC_j}I_{R_j} \quad (31)$$
$$-I_{R_S}(\frac{1}{R_SC_P} + \frac{1}{R_SC_j})$$

We define

$$Y = I_{L_P} \Rightarrow \frac{dI_{R_S}}{dt} = \frac{1}{R_SC_P}Y + \frac{1}{R_SC_j}I_{R_j}$$
$$-I_{R_S}(\frac{1}{R_SC_P} + \frac{1}{R_SC_j}) \quad (32)$$

$$\frac{V(t)}{R_{in}} - \frac{L_1}{R_{in}}\frac{dI_{L_1}}{dt} = I_{L_1} + I_{L_P}$$
$$\frac{V(t)}{R_{in}} - I_{L_1} - I_{L_P} = \frac{L_1}{R_{in}}\frac{dI_{L_1}}{dt} \quad (33)$$
$$\frac{V(t)}{L_1} - \frac{R_{in}}{L_1}I_{L_1} - \frac{R_{in}}{L_1}I_{L_P} = \frac{dI_{L_1}}{dt}$$

$$I_{L_P} - I_{R_S} = C_P[\frac{dV(t)}{dt} - R_{in}\frac{dI_{L_P}}{dt} - R_{in}\frac{dI_{L_1}}{dt}$$
$$-L_P\frac{d^2I_{L_P}}{dt^2} - \frac{I_{L_P}}{C_1}] \quad (34)$$

$$I_{L_P} - I_{R_S} = C_P[\frac{dV(t)}{dt} - R_{in}\frac{dI_{L_P}}{dt}$$
$$-R_{in}(\frac{V(t)}{L_1} - \frac{R_{in}}{L_1}I_{L_1} - \frac{R_{in}}{L_1}I_{L_P}) \quad (35)$$
$$-L_P\frac{d^2I_{L_P}}{dt^2} - \frac{I_{L_P}}{C_1}]$$

$$\begin{gathered} I_{L_P} - I_{R_S} = C_P[\frac{dV(t)}{dt} - R_{in}\frac{dI_{L_P}}{dt} - \frac{R_{in}V(t)}{L_1} \\ + \frac{R_{in}^2}{L_1} I_{L_1} + \frac{R_{in}^2}{L_1} I_{L_P} - L_P \frac{d^2 I_{L_P}}{dt^2} - \frac{I_{L_P}}{C_1}] \end{gathered} \tag{36}$$

$$\begin{gathered} -I_{L_P} + I_{R_S} + C_P\frac{dV(t)}{dt} - C_P R_{in}\frac{dI_{L_P}}{dt} \\ -\frac{C_P R_{in} V(t)}{L_1} + C_P \frac{R_{in}^2}{L_1} I_{L_1} + C_P\frac{R_{in}^2}{L_1} I_{L_P} \\ -C_P L_P \frac{d^2 I_{L_P}}{dt^2} - \frac{C_P I_{L_P}}{C_1} = 0 \end{gathered} \tag{37}$$

$$\begin{gathered} -C_P L_P \frac{d^2 I_{L_P}}{dt^2} - C_P R_{in}\frac{dI_{L_P}}{dt} + I_{L_P}[C_P\frac{R_{in}^2}{L_1} \\ -\frac{C_P}{C_1} - 1] + I_{R_S} + C_P\frac{R_{in}^2}{L_1} I_{L_1} - \frac{C_P R_{in} V(t)}{L_1} \\ + C_P\frac{dV(t)}{dt} = 0 \end{gathered} \tag{38}$$

We define:

$$\begin{gathered} Y = I_{L_P}; X = \frac{dI_{L_P}}{dt}; \frac{dX}{dt} = \frac{d^2 I_{L_P}}{dt^2} \\ \frac{dY}{dt} = \frac{dI_{L_P}}{dt} = X \end{gathered} \tag{39}$$

Then we get the expression:

$$\begin{gathered} -C_P L_P\frac{dX}{dt} - C_P R_{in} X + Y[C_P\frac{R_{in}^2}{L_1} - \frac{C_P}{C_1} - 1] \\ + I_{R_S} + C_P\frac{R_{in}^2}{L_1} I_{L_1} - \frac{C_P R_{in} V(t)}{L_1} + C_P\frac{dV(t)}{dt} = 0 \end{gathered} \tag{40}$$

$$\begin{gathered} C_P L_P\frac{dX}{dt} = -C_P R_{in} X + Y[C_P\frac{R_{in}^2}{L_1} - \frac{C_P}{C_1} - 1] \\ + I_{R_S} + C_P\frac{R_{in}^2}{L_1} I_{L_1} - \frac{C_P R_{in} V(t)}{L_1} + C_P\frac{dV(t)}{dt} \end{gathered} \tag{41}$$

$$\begin{gathered} \frac{dX}{dt} = -\frac{R_{in}}{L_P} X + Y[\frac{R_{in}^2}{L_1 L_P} - \frac{1}{C_1 L_P} - \frac{1}{C_P L_P}] \\ + I_{R_S}\frac{1}{C_P L_P} + \frac{R_{in}^2}{L_1 L_P} I_{L_1} - \frac{R_{in} V(t)}{L_1 L_P} + \frac{1}{L_P}\frac{dV(t)}{dt} \end{gathered} \tag{42}$$

$$\begin{gathered} \frac{dY}{dt} = X; \frac{dI_{L_1}}{dt} = \frac{V(t)}{L_1} - \frac{R_{in}}{L_1} I_{L_1} - \frac{R_{in}}{L_1} Y \\ \frac{dI_{R_j}}{dt} = \frac{1}{C_j R_j} I_{R_S} - \frac{1}{C_j R_j} I_{R_j} \end{gathered} \tag{43}$$

$$\frac{dI_{R_S}}{dt} = \frac{1}{R_S C_P} Y + \frac{1}{R_S C_j} I_{R_j} - I_{R_S}\frac{1}{R_S}(\frac{1}{C_P} + \frac{1}{C_j}) \tag{44}$$

We have five variables in our system: $X, Y, I_{L_1}, I_{R_j}, I_{R_S}$ and we can represent our system as the following set of differential equations matrix representation.

$$\begin{gathered} \Xi_{11} = -\frac{R_{in}}{L_P}; \Xi_{12} = \frac{R_{in}^2}{L_1 L_P} - \frac{1}{C_1 L_P} - \frac{1}{C_P L_P} \\ \Xi_{12} = \frac{1}{L_P}(\frac{R_{in}^2}{L_1} - \frac{1}{C_1} - \frac{1}{C_P}); \Xi_{13} = \frac{R_{in}^2}{L_1 L_P}; \Xi_{14} = 0 \end{gathered} \tag{45}$$

$$\begin{pmatrix} \frac{dX}{dt} \\ \frac{dY}{dt} \\ \frac{dI_{L_1}}{dt} \\ \frac{dI_{R_j}}{dt} \\ \frac{dI_{R_S}}{dt} \end{pmatrix} = \begin{pmatrix} \Xi_{11} & \dots & \Xi_{1n} \\ \vdots & \ddots & \vdots \\ \Xi_{m1} & \dots & \Xi_{mn} \end{pmatrix}_{n=m=5} \begin{pmatrix} X \\ Y \\ I_{L_1} \\ I_{R_j} \\ I_{R_S} \end{pmatrix} + \begin{pmatrix} -\frac{R_{in}}{L_1 L_P} \\ 0 \\ \frac{1}{L_1} \\ 0 \\ 0 \end{pmatrix} V(t) + \begin{pmatrix} \frac{1}{L_P} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \frac{dV(t)}{dt} \tag{46}$$

$$\begin{gathered} \Xi_{15} = \frac{1}{C_P L_P}; \Xi_{21} = 1; \Xi_{22} = \Xi_{23} = \Xi_{24} = \Xi_{25} = 0 \\ \Xi_{31} = 0; \Xi_{32} = -\frac{R_{in}}{L_1}; \Xi_{33} = -\frac{R_{in}}{L_1} \end{gathered} \tag{47}$$

$$\begin{gathered} \Xi_{34} = \Xi_{35} = 0; \Xi_{41} = \Xi_{42} = \Xi_{43} = 0 \\ \Xi_{44} = -\frac{1}{C_j R_j}; \Xi_{45} = \frac{1}{C_j R_j}; \Xi_{51} = 0; \Xi_{52} = \frac{1}{R_S C_P} \end{gathered} \tag{48}$$

$$\Xi_{53} = 0; \Xi_{54} = \frac{1}{R_S C_j}; \Xi_{55} = -\frac{1}{R_S}(\frac{1}{C_P} + \frac{1}{C_j}) \tag{49}$$

We consider RFin signal

$$V(t) = A_0 + f(t) \tag{50}$$

$$V(t) = A_0 + f(t); |f(t)| < 1 \,\&\, A_0 \gg |f(t)| \tag{51}$$

$$V(t)|_{A_0 \gg |f(t)|}; V(t)|_{A_0 \gg |f(t)|} = A_0 + f(t) \approx A_0 \tag{52}$$

$$\frac{dV(t)}{dt}|_{A_0 \gg |f(t)|} = \frac{df(t)}{dt} \to \varepsilon \tag{53}$$

We can present our matrix representation: $\varepsilon \to 0$. Due to parasitic delay elements in Schottky equivalent circuit, τ_1 for the current flow through Schottky diode's package parasitic inductance (L_P) and τ_2 for the current flow through Schottky diode's parasitic resistance (R_S), we get the following transformation [3] [4].

$$\begin{gathered} Y(t) = I_{L_P}(t) \to Y(t-\tau_1) = I_{L_P}(t-\tau_1) \\ I_{R_S}(t) \to I_{R_S}(t-\tau_2) \end{gathered} \tag{54}$$

and $X(t) = \frac{dI_{L_P}(t)}{dt}; I_{L_1}(t); I_{R_j}(t)$. We consider no delay effects on $\frac{dY}{dt} = \frac{dI_{L_P}}{dt}; \frac{dI_{R_S}}{dt}$. To find equilibrium points (fixed points) of the RFID tag detector, we define

$$\begin{gathered} \lim_{t\to\infty} Y(t-\tau_1) = Y(t); \lim_{t\to\infty} I_{L_P}(t-\tau_1) = I_{L_P}(t) \\ \lim_{t\to\infty} I_{R_S}(t-\tau_2) = I_{R_S}(t) \end{gathered} \tag{55}$$

$$\begin{pmatrix} \frac{dX}{dt} \\ \frac{dY}{dt} \\ \frac{dI_{L_1}}{dt} \\ \frac{dI_{R_j}}{dt} \\ \frac{dI_{R_S}}{dt} \end{pmatrix} = \begin{pmatrix} \Xi_{11} & \dots & \Xi_{1n} \\ \vdots & \ddots & \vdots \\ \Xi_{m1} & \dots & \Xi_{mn} \end{pmatrix}_{n=m=5} \begin{pmatrix} X \\ Y \\ I_{L_1} \\ I_{R_j} \\ I_{R_S} \end{pmatrix} + \begin{pmatrix} -\frac{R_{in}}{L_1 L_P} \\ 0 \\ \frac{1}{L_1} \\ 0 \\ 0 \end{pmatrix} A_0 + \varepsilon \tag{56}$$

In equilibrium points (fixed points)

$$\begin{gathered} \frac{dY}{dt} = \frac{dI_{L_P}}{dt} = 0; \frac{dI_{R_S}}{dt} = 0 \, \forall \, t \gg \tau_1, t \gg \tau_2 \\ \exists (t-\tau_1) \approx t, (t-\tau_2) \approx t, t \to \infty \end{gathered} \tag{57}$$

We get five equations:

$$-\frac{R_{in}}{L_P}X^* + Y^*[\frac{R_{in}^2}{L_1 L_P} - \frac{1}{C_1 L_P} - \frac{1}{C_P L_P}] + I_{R_S}^* \frac{1}{C_P L_P} + \frac{R_{in}^2}{L_1 L_P} I_{L_1}^* - \frac{R_{in} V(t)}{L_1 L_P} + \frac{1}{L_P}\frac{dV(t)}{dt} = 0 \quad (58)$$

$$X^* = 0; \frac{V(t)}{L_1} - \frac{R_{in}}{L_1} I_{L_1}^* - \frac{R_{in}}{L_1} Y^* = 0 \quad \frac{1}{C_j R_j} I_{R_S}^* - \frac{1}{C_j R_j} I_{R_j}^* = 0 \quad (59)$$

$$\frac{1}{R_s C_P} Y^* + \frac{1}{R_s C_j} I_{R_j}^* - I_{R_s}^* (\frac{1}{C_P} + \frac{1}{C_j}) = 0 \quad (60)$$

Since $X^* = 0$ then

$$Y^*[\frac{R_{in}^2}{L_1 L_P} - \frac{1}{C_1 L_P} - \frac{1}{C_P L_P}] + I_{R_S}^* \frac{1}{C_P L_P} + \frac{R_{in}^2}{L_1 L_P} I_{L_1}^* - \frac{R_{in} V(t)}{L_1 L_P} + \frac{1}{L_P}\frac{dV(t)}{dt} = 0 \quad (61)$$

$$\frac{V(t)}{L_1} - \frac{R_{in}}{L_1} I_{L_1}^* - \frac{R_{in}}{L_1} Y^* = 0 \Rightarrow Y^* = \frac{V(t)}{R_{in}} - I_{L_1}^* \quad (62)$$

$$\frac{1}{R_S C_P}(\frac{V(t)}{R_{in}} - I_{L_1}^*) + \frac{1}{R_S C_j} I_{R_j}^* - I_{R_S}^* \frac{1}{R_S}(\frac{1}{C_P} + \frac{1}{C_j}) = 0 \quad (63)$$

$$(\frac{V(t)}{R_{in}} - I_{L_1}^*)[\frac{R_{in}^2}{L_1 L_P} - \frac{1}{C_1 L_P} - \frac{1}{C_P L_P}] + I_{R_S}^* \frac{1}{C_P L_P} + \frac{R_{in}^2}{L_1 L_P} I_{L_1}^* - \frac{R_{in} V(t)}{L_1 L_P} + \frac{1}{L_P}\frac{dV(t)}{dt} = 0 \quad (64)$$

We get three equations:

$$\frac{1}{C_j R_j} I_{R_S}^* - \frac{1}{C_j R_j} I_{R_j}^* = 0 \quad (65)$$

$$\frac{1}{R_S C_P}(\frac{V(t)}{R_{in}} - I_{L_1}^*) + \frac{1}{R_S C_j} I_{R_j}^* - I_{R_S}^* \frac{1}{R_S}(\frac{1}{C_P} + \frac{1}{C_j}) = 0 \quad (66)$$

$$(\frac{V(t)}{R_{in}} - I_{L_1}^*)[\frac{R_{in}^2}{L_1 L_P} - \frac{1}{C_1 L_P} - \frac{1}{C_P L_P}] + I_{R_S}^* \frac{1}{C_P L_P} + \frac{R_{in}^2}{L_1 L_P} I_{L_1}^* - \frac{R_{in} V(t)}{L_1 L_P} + \frac{1}{L_P}\frac{dV(t)}{dt} = 0 \quad (67)$$

$$\frac{1}{C_j R_j} I_{R_S}^* - \frac{1}{C_j R_j} I_{R_j}^* = 0 \Rightarrow I_{R_j}^* = I_{R_S}^* \quad (68)$$

We get two equations:

$$\frac{1}{R_S C_P}(\frac{V(t)}{R_{in}} - I_{L_1}^*) + \frac{1}{R_S C_j} I_{R_S}^* - I_{R_S}^* \frac{1}{R_S}(\frac{1}{C_P} + \frac{1}{C_j}) = 0 \quad (69)$$

$$(\frac{V(t)}{R_{in}} - I_{L_1}^*)[\frac{R_{in}^2}{L_1 L_P} - \frac{1}{C_1 L_P} - \frac{1}{C_P L_P}] + I_{R_S}^* \frac{1}{C_P L_P} + \frac{R_{in}^2}{L_1 L_P} I_{L_1}^* - \frac{R_{in} V(t)}{L_1 L_P} + \frac{1}{L_P}\frac{dV(t)}{dt} = 0 \quad (70)$$

By mathematic manipulation, we get the following two equations:

$$\frac{V(t)}{R_{in}} - I_{L_1}^* - I_{R_S}^* = 0 \Rightarrow I_{R_S}^* = \frac{V(t)}{R_{in}} - I_{L_1}^* \quad (71)$$

$$I_{L_1}^*(\frac{1}{C_1} + \frac{1}{C_P}) + I_{R_S}^* \frac{1}{C_P} + V(t)\{\frac{1}{R_{in}}[\frac{R_{in}^2}{L_1} - \frac{1}{C_1} - \frac{1}{C_P}] - \frac{R_{in}}{L_1}\} + \frac{dV(t)}{dt} = 0 \quad (72)$$

We define for simplicity:

$$\Omega = \frac{1}{R_{in}}[\frac{R_{in}^2}{L_1} - \frac{1}{C_1} - \frac{1}{C_P}] - \frac{R_{in}}{L_1} \quad (73)$$

$$I_{R_S}^* = \frac{V(t)}{R_{in}} - I_{L_1}^*; I_{L_1}^*(\frac{1}{C_1} + \frac{1}{C_P}) + I_{R_S}^* \frac{1}{C_P} + V(t)\Omega + \frac{dV(t)}{dt} = 0 \quad (74)$$

$$I_{L_1}^*(\frac{1}{C_1} + \frac{1}{C_P}) + (\frac{V(t)}{R_{in}} - I_{L_1}^*)\frac{1}{C_P} + V(t)\Omega + \frac{dV(t)}{dt} = 0 \quad I_{L_1}^* \frac{1}{C_1} + V(t)[\frac{1}{R_{in} C_P} + \Omega] + \frac{dV(t)}{dt} = 0 \quad (75)$$

$$I_{L_1}^* \frac{1}{C_1} + V(t)[\frac{1}{R_{in} C_P} + \Omega] + \frac{dV(t)}{dt} = 0 \quad I_{L_1}^* = -C_1\{V(t)[\frac{1}{R_{in} C_P} + \Omega] + \frac{dV(t)}{dt}\} \quad (76)$$

$$I_{R_S}^* = \frac{V(t)}{R_{in}} + C_1\{V(t)[\frac{1}{R_{in} C_P} + \Omega] + \frac{dV(t)}{dt}\} = V(t)\{\frac{1}{R_{in}} + C_1[\frac{1}{R_{in} C_P} + \Omega]\} + C_1 \frac{dV(t)}{dt} \quad (77)$$

$$\Omega_1 = \frac{1}{R_{in}} + C_1[\frac{1}{R_{in} C_p} + \Omega]; I_{R_S}^* = V(t)\Omega_1 + C_1 \frac{dV(t)}{dt} \quad (78)$$

$$I_{R_j}^* = I_{R_S}^* \Rightarrow I_{R_j}^* = V(t)\Omega_1 + C_1 \frac{dV(t)}{dt}; X^* = 0 \quad (79)$$

$$Y^* = \frac{V(t)}{R_{in}} - I_{L_1}^* \quad Y^* = V(t)\{\frac{1}{R_{in}} + C_1[\frac{1}{R_{in} C_P} + \Omega]\} + C_1 \frac{dV(t)}{dt} \quad (80)$$

We can summery our system fixed points in the next tables:

Fixed point coordinates $E^*(X^*, Y^*, I_{L_1}^*, I_{R_j}^*, I_{R_S}^*)$	Fixed points expression $V(t) = A_0 + f(t)$ $\lvert f(t)\rvert < 1; A_0 \gg \lvert f(t)\rvert$
X^*	0
Y^*	$V(t)\{\frac{1}{R_{in}} + C_1[\frac{1}{R_{in} C_P} + \Omega]\} + C_1 \frac{dV(t)}{dt}$
$I_{L_1}^*$	$-C_1\{V(t)[\frac{1}{R_{in} C_p} + \Omega] + \frac{dV(t)}{dt}\}$
$I_{R_j}^*$	$V(t)\Omega_1 + C_1 \frac{dV(t)}{dt}$
$I_{R_S}^*$	$V(t)\{\frac{1}{R_{in}} + C_1[\frac{1}{R_{in} C_p} + \Omega]\} + C_1 \frac{dV(t)}{dt}$

.

Table. 1a. RFID tag receiver detector system fixed points.

Fixed point coordinates $E^*(X^*, Y^*, I_{L_1}^*, I_{R_j}^*, I_{R_S}^*)$	$V(t)\rvert_{A_0 \gg \lvert f(t)\rvert} = A_0 + f(t) \approx A_0$ $\frac{dV(t)}{dt}\rvert_{A_0 \gg \lvert f(t)\rvert} = \frac{df(t)}{dt} \to \varepsilon$
X^*	0
Y^*	$A_0\{\frac{1}{R_{in}} + C_1[\frac{1}{R_{in} C_P} + \Omega]\}$
$I_{L_1}^*$	$-C_1 A_0[\frac{1}{R_{in} C_P} + \Omega]$
$I_{R_j}^*$	$A_0 \Omega_1$
$I_{R_S}^*$	$A_0\{\frac{1}{R_{in}} + C_1[\frac{1}{R_{in} C_P} + \Omega]\}$

.

Table. 1b. RFID tag receiver detector system fixed points.

III. RFID TAG RECEIVER DETECTOR STABILITY ANALYSIS UNDER DELAY VARIABLES IN TIME

We can check our RFID tag receiver detector system stability for the following cases.

$$(A)\,\tau_1=\tau;\tau_2=0\,(B)\,\tau_1=0;\tau_2=\tau\,(C)\,\tau_1=\tau_2=\tau \quad (81)$$

Stability analysis: The standard local stability analysis about any one of the equilibrium points of the RFID tag detector system consists in adding to coordinate $[X,Y,I_{L_1},I_{R_j},I_{R_S}]$ arbitrarily small increments of exponential form $[x,y,i_{L_1},i_{R_j},i_{R_s}]e^{\lambda t}$ and retaining the first order terms in $X,Y,I_{L_1},I_{R_j},I_{R_S}$. The system of five homogeneous equations leads to a polynomial characteristic equation in the eigenvalues. The polynomial characteristic equations accept by set the below currents and currents derivative with respect to time into RFID tag detector system equations. RFID tag detector system fixed values with arbitrarily small increments of exponential form $[x,y,i_{L_1},i_{R_j},i_{R_s}]e^{\lambda t}$ are: $j=0$ (first fixed point), $j=1$ (second fixed point), $j=2$ (third fixed point), etc.

$$\begin{aligned}&X(t)=X^{(j)}+xe^{\lambda t};Y(t)=Y^{(j)}+ye^{\lambda t}\\&Y(t-\tau_1)=Y^{(j)}+ye^{\lambda(t-\tau_1)}\\&I_{L_1}(t)=I_{L_1}^{(j)}+i_{L_1}e^{\lambda t}\end{aligned}\quad (82)$$

$$\begin{aligned}&I_{R_j}(t)=I_{R_j}^{(j)}+i_{R_j}e^{\lambda t};I_{R_S}(t)=I_{R_S}^{(j)}+i_{R_S}e^{\lambda t}\\&I_{R_S}(t-\tau_2)=I_{R_S}^{(j)}+i_{R_S}e^{\lambda(t-\tau_2)}\end{aligned}\quad (83)$$

We choose these expressions for ourselves $X(t),Y(t),I_{L_1}(t),I_{R_j}(t),I_{R_S}(t)$ as a small displacement $[x,y,i_{L_1},i_{R_j},i_{R_S}]$ from the RFID tag detector system fixed points in time $t=0$.

$$\begin{aligned}&X(t=0)=X^{(j)}+x;Y(t=0)=Y^{(j)}+y\\&I_{L_1}(t=0)=I_{L_1}^{(j)}+i_{L_1};I_{R_j}(t=0)=I_{R_j}^{(j)}+i_{R_j}\end{aligned}\quad (84)$$

$$I_{R_S}(t=0)=I_{R_S}^{(j)}+i_{R_S} \quad (85)$$

For $\lambda<0,t>0$ the selected fixed point is stable otherwise $\lambda>0,t>0$ is unstable. Our system tends to the selected fixed point exponentially for $\lambda<0,t>0$ otherwise go away from the selected fixed point exponentially. Eigenvalue λ parameter is established if the fixed point is stable or unstable; additionally, his absolute value $|\lambda|$ establishes the speed of flow toward or away from the selected fixed point (Yuri, 1995; Jack and Huseyin, 1991) [5] [6]. The speeds of flow toward or away from the selected fixed point for Schottky detector system currents and currents derivatives with respect to time are as follow.

$$\begin{aligned}\frac{dX(t)}{dt}&=\lim_{\Delta t\to\infty}\frac{X(t+\Delta t)-X(t)}{\Delta t}\\&=\lim_{\Delta t\to\infty}\frac{X^{(j)}+xe^{\lambda(t+\Delta t)}-[X^{(j)}+xe^{\lambda t}]}{\Delta t}\\&=\lambda xe^{\lambda t}\end{aligned}\quad (86)$$

$$\begin{aligned}\frac{dY(t)}{dt}&=\lim_{\Delta t\to\infty}\frac{Y(t+\Delta t)-Y(t)}{\Delta t}\\&=\lim_{\Delta t\to\infty}\frac{Y^{(j)}+ye^{\lambda(t+\Delta t)}-[Y^{(j)}+ye^{\lambda t}]}{\Delta t}\\&=\lambda ye^{\lambda t}\end{aligned}\quad (87)$$

$$\begin{aligned}&\frac{dI_{L_1}(t)}{dt}=\lambda i_{L_1}e^{\lambda t};\frac{dI_{R_j}(t)}{dt}=\lambda i_{R_j}e^{\lambda t}\\&\frac{dI_{R_S}(t)}{dt}=\lambda i_{R_S}e^{\lambda t};\frac{dY(t-\tau_1)}{dt}=\lambda ye^{\lambda t}e^{-\lambda\tau_1}\end{aligned}\quad (88)$$

$$\frac{dI_{R_S}(t-\tau_2)}{dt}=\lambda i_{R_S}e^{\lambda t}e^{-\lambda\tau_2} \quad (89)$$

First, we take Schottky detector variable $X,Y,I_{L_1},I_{R_j},I_{R_s}$ differential equations and adding to coordinate $[X,Y,I_{L_1},I_{R_j},I_{R_s}]$ arbitrarily small increments of exponential terms $[x,y,i_{L_1},i_{R_j},i_{R_s}]e^{\lambda t}$ and retaining the first order terms in $x,y,i_{L_1},i_{R_j},i_{R_s}$ $(V(t)\to\varepsilon;\frac{dV(t)}{dt}\to\varepsilon)$

$$\begin{aligned}&E^*(X^*,Y^*,I_{L_1}^*,I_{R_j}^*,I_{R_S}^*)=(0,0,0,0,0)\\&X^{(j=0)}=0;Y^{(j=0)}=0;I_{L_1}^{(j=0)}=0\\&I_{R_j}^{(j=0)}=0;I_{R_S}^{(j=0)}=0\end{aligned}\quad (90)$$

$$\begin{aligned}&\lambda_1=-\frac{R_{in}}{L_P}+\frac{y}{x}[\frac{R_{in}^2}{L_1L_P}-\frac{1}{C_1L_P}-\frac{1}{C_PL_P}]\\&+\frac{i_{R_S}}{x}\frac{1}{C_PL_P}+\frac{R_{in}^2}{L_1L_P}\frac{i_{L_1}}{x}\\&\lambda_2=\frac{x}{y};\lambda_3=-\frac{R_{in}}{L_1}-\frac{R_{in}}{L_1}\frac{y}{i_{L_1}}\end{aligned}\quad (91)$$

$$\begin{aligned}&\lambda_4=\frac{1}{C_jR_j}\frac{i_{R_S}}{i_{R_j}}-\frac{1}{C_jR_j}\\&\lambda_5=\frac{1}{R_SC_P}\frac{y}{i_{R_S}}+\frac{1}{R_SC_j}\frac{i_{R_j}}{i_{R_S}}-\frac{1}{R_S}(\frac{1}{C_P}+\frac{1}{C_j})\end{aligned}\quad (92)$$

We consider

$$\begin{aligned}&\frac{y}{x}\approx 1;\frac{i_{R_S}}{x}\approx 1;\frac{i_{L_1}}{x}\approx 1;\frac{x}{y}\approx 1;\frac{y}{i_{L_1}}\approx 1\\&\frac{i_{R_S}}{i_{R_j}}\approx 1;\frac{y}{i_{R_S}}\approx 1;\frac{i_{R_j}}{i_{R_S}}\approx 1\end{aligned}\quad (93)$$

$$\begin{aligned}&\lambda_1=\frac{2R_{in}^2}{L_1L_P}-[\frac{1}{C_1L_P}+\frac{R_{in}}{L_P}];\lambda_2=1\\&\lambda_3=-\frac{2R_{in}}{L_1}<0;\lambda_4=0;\lambda_5=0\end{aligned}\quad (94)$$

$$\frac{2R_{in}^2}{L_1L_P}>[\frac{1}{C_1L_P}+\frac{R_{in}}{L_P}] \quad (95)$$

$$\begin{aligned}&\frac{2R_{in}^2}{L_1L_P}>[\frac{1}{C_1L_P}+\frac{R_{in}}{L_P}]\\&\frac{2R_{in}^2}{L_1}>\frac{1+R_{in}C_1}{C_1}\Rightarrow\lambda_1>0\end{aligned}\quad (96)$$

$$\frac{2R_{in}^2}{L_1}<\frac{1+R_{in}C_1}{C_1}\Rightarrow\lambda_1<0 \quad (97)$$

$$\frac{2R_{in}^2}{L_1}=\frac{1+R_{in}C_1}{C_1}\Rightarrow\lambda_1=0 \quad (98)$$

We can see that our fixed point is a saddle node. We define $Y(t-\tau_1)=Y^{(j)}+ye^{\lambda(t-\tau_1)};I_{R_S}(t-\tau_2)=I_{R_S}^{(j)}+i_{R_S}e^{\lambda(t-\tau_2)}$ then we get five delayed differential equations with respect to coordinates $[X,Y,I_{L_1},I_{R_j},I_{R_s}]$ arbitrarily small increments of exponential$[x,y,i_{L_1},i_{R_j},i_{R_s}]e^{\lambda t}$. We consider no delay effects on $\frac{dY(t)}{dt};\frac{dI_{R_S}(t)}{dt}$. We get the following equations:

Time (t)	$\lambda < 0$
$t = 0$	$X(t=0) = X^{(j)} + x; Y(t=0) = Y^{(j)} + y$ $I_{L_1}(t=0) = I_{L_1}^{(j)} + i_{L_1}$ $I_{R_j}(t=0) = I_{R_j}^{(j)} + i_{R_j}$ $I_{R_S}(t=0) = I_{R_S}^{(j)} + i_{R_S}$
$t > 0$	$X(t) = X^{(j)} + xe^{-\|\lambda\|t}; Y(t) = Y^{(j)} + ye^{-\|\lambda\|t}$ $I_{L_1}(t) = I_{L_1}^{(j)} + i_{L_1}e^{-\|\lambda\|t}$ $I_{R_j}(t) = I_{R_j}^{(j)} + i_{R_j}e^{-\|\lambda\|t}$ $I_{R_s}(t) = I_{R_s}^{(j)} + i_{R_s}e^{-\|\lambda\|t}$
$t > 0$ $t \to \infty$	$X(t \to \infty) = X^{(j)}; Y(t \to \infty) = Y^{(j)}$ $I_{L_1}(t \to \infty) = I_{L_1}^{(j)}; I_{R_j}(t \to \infty) = I_{R_j}^{(j)}$ $I_{R_S}(t \to \infty) = I_{R_S}^{(j)}$

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Table. 2a. RFID tag receiver detector system variables for negative eigenvalue ($\lambda < 0$).

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Time (t)	$\lambda > 0$
$t = 0$	$X(t=0) = X^{(j)} + x; Y(t=0) = Y^{(j)} + y$ $I_{L_1}(t=0) = I_{L_1}^{(j)} + i_{L_1}$ $I_{R_j}(t=0) = I_{R_j}^{(j)} + i_{R_j}$ $I_{R_S}(t=0) = I_{R_S}^{(j)} + i_{R_S}$
$t > 0$	$X(t) = X^{(j)} + xe^{\|\lambda\|t}; Y(t) = Y^{(j)} + ye^{\|\lambda\|t}$ $I_{L_1}(t) = I_{L_1}^{(j)} + i_{L_1}e^{\|\lambda\|t}$ $I_{R_j}(t) = I_{R_j}^{(j)} + i_{R_j}e^{\|\lambda\|t}$ $I_{R_s}(t) = I_{R_s}^{(j)} + i_{R_s}e^{\|\lambda\|t}$
$t > 0$ $t \to \infty$	$X(t \to \infty) = xe^{\|\lambda\|t}; Y(t \to \infty) = ye^{\|\lambda\|t}$ $I_{L_1}(t \to \infty) = i_{L_1}e^{\|\lambda\|t}$ $I_{R_j}(t \to \infty) = i_{R_j}e^{\|\lambda\|t}$ $I_{R_S}(t \to \infty) = i_{R_s}e^{\|\lambda\|t}$

.

Table. 2b. RFID tag receiver detector system variables for negative eigenvalue ($\lambda > 0$).

$$\begin{aligned}&\lambda xe^{\lambda t} = -\tfrac{R_{in}}{L_P}[X^{(j)} + xe^{\lambda t}] + [Y^{(j)} \\ &+ye^{\lambda(t-\tau_1)}][\tfrac{R_{in}^2}{L_1L_P} - \tfrac{1}{C_1L_P} - \tfrac{1}{C_PL_P}] \\ &+[I_{R_S}^{(j)} + i_{R_S}e^{\lambda(t-\tau_2)}]\tfrac{1}{C_PL_P} \\ &+\tfrac{R_{in}^2}{L_1L_P}[I_{L_1}^{(j)} + i_{L_1}e^{\lambda t}] - \tfrac{R_{in}V(t)}{L_1L_P} + \tfrac{1}{L_P}\tfrac{dV(t)}{dt} \\ &V(t), \tfrac{dV(t)}{dt} \to \varepsilon\end{aligned} \tag{99}$$

$$\begin{aligned}&\lambda xe^{\lambda t} = -\tfrac{R_{in}}{L_P}X^{(j)} - \tfrac{R_{in}}{L_P}xe^{\lambda t} \\ &+Y^{(j)}[\tfrac{R_{in}^2}{L_1L_P} - \tfrac{1}{C_1L_P} - \tfrac{1}{C_PL_P}] \\ &+y[\tfrac{R_{in}^2}{L_1L_P} - \tfrac{1}{C_1L_P} - \tfrac{1}{C_PL_P}]e^{\lambda(t-\tau_1)} + I_{R_S}^{(j)}\tfrac{1}{C_PL_P} \\ &+i_{R_S}\tfrac{1}{C_PL_P}e^{\lambda(t-\tau_2)} + \tfrac{R_{in}^2}{L_1L_P}I_{L_1}^{(j)} + \tfrac{R_{in}^2}{L_1L_P}i_{L_1}e^{\lambda t}\end{aligned} \tag{100}$$

$$\begin{aligned}&\lambda xe^{\lambda t} = -\tfrac{R_{in}}{L_P}X^{(j)} + Y^{(j)}[\tfrac{R_{in}^2}{L_1L_P} - \tfrac{1}{C_1L_P} \\ &-\tfrac{1}{C_PL_P}] + I_{R_S}^{(j)}\tfrac{1}{C_PL_P} + \tfrac{R_{in}^2}{L_1L_P}I_{L_1}^{(j)} - \tfrac{R_{in}}{L_P}xe^{\lambda t} \\ &+y[\tfrac{R_{in}^2}{L_1L_P} - \tfrac{1}{C_1L_P} - \tfrac{1}{C_PL_P}]e^{\lambda(t-\tau_1)} \\ &+i_{R_S}\tfrac{1}{C_PL_P}e^{\lambda(t-\tau_2)} + \tfrac{R_{in}^2}{L_1L_P}i_{L_1}e^{\lambda t}\end{aligned} \tag{101}$$

At fixed point:

$$\begin{aligned}&-\tfrac{R_{in}}{L_P}X^{(j)} + Y^{(j)}[\tfrac{R_{in}^2}{L_1L_P} - \tfrac{1}{C_1L_P} - \tfrac{1}{C_PL_P}] \\ &+I_{R_S}^{(j)}\tfrac{1}{C_PL_P} + \tfrac{R_{in}^2}{L_1L_P}I_{L_1}^{(j)} = 0\end{aligned} \tag{102}$$

$$\begin{aligned}&-xe^{\lambda t}[\lambda + \tfrac{R_{in}}{L_P}] + y[\tfrac{R_{in}^2}{L_1L_P} - \tfrac{1}{C_1L_P} - \tfrac{1}{C_PL_P}]e^{\lambda(t-\tau_1)} \\ &+i_{R_S}\tfrac{1}{C_PL_P}e^{\lambda(t-\tau_2)} + \tfrac{R_{in}^2}{L_1L_P}i_{L_1}e^{\lambda t} = 0\end{aligned} \tag{103}$$

$$\frac{dY}{dt} = X \Rightarrow \lambda ye^{\lambda t} = X^{(j)} + xe^{\lambda t} \tag{104}$$

At fixed point

$$X^{(j)} = 0 \Rightarrow -x + \lambda y = 0 \tag{105}$$

$$\begin{aligned}&\lambda i_{L_1}e^{\lambda t} = \tfrac{V(t)}{L_1} - \tfrac{R_{in}}{L_1}[I_{L_1}^{(j)} + i_{L_1}e^{\lambda t}] \\ &-\tfrac{R_{in}}{L_1}[Y^{(j)} + ye^{\lambda(t-\tau_1)}]; V(t) \to \varepsilon\end{aligned} \tag{106}$$

$$\begin{aligned}&\lambda i_{L_1}e^{\lambda t} = -\tfrac{R_{in}}{L_1}I_{L_1}^{(j)} - \tfrac{R_{in}}{L_1}Y^{(j)} \\ &-i_{L_1}\tfrac{R_{in}}{L_1}e^{\lambda t} - y\tfrac{R_{in}}{L_1}e^{\lambda(t-\tau_1)}\end{aligned} \tag{107}$$

At fixed point

$$-\frac{R_{in}}{L_1}I_{L_1}^{(j)} - \frac{R_{in}}{L_1}Y^{(j)} = 0 \tag{108}$$

$$-\lambda i_{L_1}e^{\lambda t} - i_{L_1}\frac{R_{in}}{L_1}e^{\lambda t} - y\frac{R_{in}}{L_1}e^{\lambda(t-\tau_1)} = 0 \tag{109}$$

$$\begin{aligned}&\lambda i_{R_j}e^{\lambda t} = \tfrac{1}{C_jR_j}[I_{R_S}^{(j)} + i_{R_S}e^{\lambda(t-\tau_2)}] \\ &-\tfrac{1}{C_jR_j}[I_{R_j}^{(j)} + i_{R_j}e^{\lambda t}]\end{aligned} \tag{110}$$

$$\begin{aligned}&-\lambda i_{R_j}e^{\lambda t} - i_{R_j}\tfrac{1}{C_jR_j}e^{\lambda t} + i_{R_S}\tfrac{1}{C_jR_j}e^{\lambda(t-\tau_2)} \\ &+\tfrac{1}{C_jR_j}I_{R_S}^{(j)} - \tfrac{1}{C_jR_j}I_{R_j}^{(j)} = 0\end{aligned} \tag{111}$$

At fixed point

$$\frac{1}{C_jR_j}I_{R_s}^{(j)} - \frac{1}{C_jR_j}I_{R_j}^{(j)} = 0 \tag{112}$$

$$-i_{R_j}e^{\lambda t}[\lambda + \frac{1}{C_jR_j}] + i_{R_s}\frac{1}{C_jR_j}e^{\lambda(t-\tau_2)} = 0 \tag{113}$$

$$\begin{aligned}&\lambda i_{R_S}e^{\lambda t} = \tfrac{1}{R_SC_P}[Y^{(j)} + ye^{\lambda(t-\tau_1)}] \\ &+\tfrac{1}{R_SC_j}[I_{R_j}^{(j)} + i_{R_j}e^{\lambda t}] \\ &-[I_{R_S}^{(j)} + i_{R_S}e^{\lambda(t-\tau_2)}]\tfrac{1}{R_S}(\tfrac{1}{C_P} + \tfrac{1}{C_j})\end{aligned} \tag{114}$$

$$\begin{aligned}&\lambda i_{R_S}e^{\lambda t} = \tfrac{1}{R_SC_P}Y^{(j)} + y\tfrac{1}{R_SC_P}e^{\lambda(t-\tau_1)} \\ &+\tfrac{1}{R_SC_j}I_{R_j}^{(j)} + i_{R_j}\tfrac{1}{R_SC_j}e^{\lambda t} - I_{R_S}^{(j)}\tfrac{1}{R_S}(\tfrac{1}{C_P} + \tfrac{1}{C_j}) \\ &-i_{R_S}\tfrac{1}{R_S}(\tfrac{1}{C_P} + \tfrac{1}{C_j})e^{\lambda(t-\tau_2)}\end{aligned} \tag{115}$$

$$\begin{aligned}&\lambda i_{R_S}e^{\lambda t} = \tfrac{1}{R_SC_P}Y^{(j)} + \tfrac{1}{R_SC_j}I_{R_j}^{(j)} \\ &-I_{R_S}^{(j)}\tfrac{1}{R_S}(\tfrac{1}{C_P} + \tfrac{1}{C_j}) + y\tfrac{1}{R_SC_P}e^{\lambda(t-\tau_1)} \\ &+i_{R_j}\tfrac{1}{R_SC_j}e^{\lambda t} - i_{R_S}\tfrac{1}{R_S}(\tfrac{1}{C_P} + \tfrac{1}{C_j})e^{\lambda(t-\tau_2)}\end{aligned} \tag{116}$$

At fixed point:

$$\frac{1}{R_S C_P} Y^{(j)} + \frac{1}{R_S C_j} I_{R_j}^{(j)} - I_{R_S}^{(j)} \frac{1}{R_S} (\frac{1}{C_P} + \frac{1}{C_j}) = 0 \quad (117)$$

$$\begin{aligned} & -i_{R_S} e^{\lambda t} [\lambda + \tfrac{1}{R_S} (\tfrac{1}{C_P} + \tfrac{1}{C_j}) e^{-\lambda \tau_2}] \\ & + y \tfrac{1}{R_S C_P} e^{\lambda (t - \tau_1)} + i_{R_j} \tfrac{1}{R_S C_j} e^{\lambda t} = 0 \end{aligned} \quad (118)$$

We can summarize our last results:

$$\begin{aligned} & -x[\lambda + \tfrac{R_{in}}{L_P}] + y[\tfrac{R_{in}^2}{L_1 L_P} - \tfrac{1}{C_1 L_P} - \tfrac{1}{C_P L_P}] e^{-\lambda \tau_1} \\ & + \tfrac{R_{in}^2}{L_1 L_P} i_{L_1} + i_{R_S} \tfrac{1}{C_P L_P} e^{-\lambda \tau_2} = 0; x - \lambda y = 0 \end{aligned} \quad (119)$$

$$-y \frac{R_{in}}{L_1} e^{-\lambda \tau_1} - i_{L_1} [\frac{R_{in}}{L_1} + \lambda] = 0 \quad (120)$$

$$-i_{R_j} [\lambda + \frac{1}{C_j R_j}] + i_{R_s} \frac{1}{C_j R_j} e^{-\lambda \tau_2} = 0 \quad (121)$$

$$\begin{aligned} & y \tfrac{1}{R_S C_P} e^{-\lambda \tau_1} + i_{R_j} \tfrac{1}{R_S C_j} \\ & - i_{R_S} [\lambda + \tfrac{1}{R_S} (\tfrac{1}{C_P} + \tfrac{1}{C_j}) e^{-\lambda \tau_2}] = 0 \end{aligned} \quad (122)$$

The small increments Jacobian of our RFID Schotky detector system is as follows:

$$\begin{pmatrix} \Upsilon_{11} & \dots & \Upsilon_{15} \\ \vdots & \ddots & \vdots \\ \Upsilon_{51} & \cdots & \Upsilon_{55} \end{pmatrix} \begin{pmatrix} x \\ y \\ i_{L_1} \\ i_{R_j} \\ i_{R_S} \end{pmatrix} = 0 \quad (123)$$

$$\Upsilon_{11} = -\frac{R_{in}}{L_P} - \lambda$$
$$\Upsilon_{12} = [\frac{R_{in}^2}{L_1 L_P} - \frac{1}{C_1 L_P} - \frac{1}{C_P L_P}] e^{-\lambda \tau_1}$$

$$\begin{aligned} & \Upsilon_{13} = \tfrac{R_{in}^2}{L_1 L_P}; \Upsilon_{14} = 0; \Upsilon_{15} = \tfrac{1}{C_P L_P} e^{-\lambda \tau_2} \\ & \Upsilon_{21} = 1; \Upsilon_{22} = -\lambda; \Upsilon_{23} = \Upsilon_{24} = \Upsilon_{25} = 0 \end{aligned} \quad (124)$$

$$\begin{aligned} & \Upsilon_{31} = 0; \Upsilon_{32} = -\tfrac{R_{in}}{L_1} e^{-\lambda \tau_1}; \Upsilon_{33} = -\tfrac{R_{in}}{L_1} - \lambda \\ & \Upsilon_{34} = 0; \Upsilon_{35} = 0; \Upsilon_{41} = \Upsilon_{42} = \Upsilon_{43} = 0 \end{aligned} \quad (125)$$

$$\begin{aligned} & \Upsilon_{44} = -\tfrac{1}{C_j R_j} - \lambda; \Upsilon_{45} = \tfrac{1}{C_j R_j} e^{-\lambda \tau_2} \\ & \Upsilon_{51} = 0; \Upsilon_{52} = \tfrac{1}{R_S C_P} e^{-\lambda \tau_1}; \Upsilon_{53} = 0 \end{aligned} \quad (126)$$

$$\Upsilon_{54} = \frac{1}{R_S C_j}; \Upsilon_{55} = -\frac{1}{R_S} (\frac{1}{C_P} + \frac{1}{C_j}) e^{-\lambda \tau_2} - \lambda \quad (127)$$

$$|A - \lambda I| = \begin{pmatrix} \Upsilon_{11} & \dots & \Upsilon_{15} \\ \vdots & \ddots & \vdots \\ \Upsilon_{51} & \cdots & \Upsilon_{55} \end{pmatrix}; \det|A - \lambda I| = 0 \quad (128)$$

We define for simplicity the following parameters:

$$\begin{aligned} & \sigma_1 = -\tfrac{R_{in}}{L_P}; \sigma_2 = \tfrac{R_{in}^2}{L_1 L_P} - \tfrac{1}{C_1 L_P} - \tfrac{1}{C_P L_P} \\ & \sigma_3 = \tfrac{R_{in}^2}{L_1 L_P}; \sigma_4 = \tfrac{1}{C_P L_P}; \sigma_5 = -\tfrac{R_{in}}{L_1} \end{aligned} \quad (129)$$

$$\begin{aligned} & \sigma_6 = \tfrac{1}{C_j R_j}; \sigma_7 = \tfrac{1}{R_S C_P}; \sigma_8 = \tfrac{1}{R_S C_j} \\ & \sigma_9 = -\tfrac{1}{R_S} (\tfrac{1}{C_P} + \tfrac{1}{C_j}) \end{aligned} \quad (130)$$

$$\begin{aligned} & \Upsilon_{11} = \sigma_1 - \lambda; \Upsilon_{12} = \sigma_2 e^{-\lambda \tau_1}; \Upsilon_{13} = \sigma_3; \Upsilon_{14} = 0 \\ & \Upsilon_{15} = \sigma_4 e^{-\lambda \tau_2}; \Upsilon_{21} = 1; \Upsilon_{22} = -\lambda \\ & \Upsilon_{23} = \Upsilon_{24} = \Upsilon_{25} = 0 \end{aligned} \quad (131)$$

$$\begin{aligned} & \Upsilon_{31} = 0; \Upsilon_{32} = \sigma_5 e^{-\lambda \tau_1}; \Upsilon_{33} = \sigma_5 - \lambda; \Upsilon_{34} = 0 \\ & \Upsilon_{35} = 0; \Upsilon_{41} = \Upsilon_{42} = \Upsilon_{43} = 0 \end{aligned} \quad (132)$$

$$\begin{aligned} & \Upsilon_{44} = -\sigma_6 - \lambda; \Upsilon_{45} = \sigma_6 e^{-\lambda \tau_2}; \Upsilon_{51} = 0 \\ & \Upsilon_{52} = \sigma_7 e^{-\lambda \tau_1}; \Upsilon_{53} = 0; \Upsilon_{54} = \sigma_8 \\ & \Upsilon_{55} = \sigma_9 e^{-\lambda \tau_2} - \lambda \end{aligned} \quad (133)$$

We need to find $D(\tau_1, \tau_2)$ for the following cases: (A) $\tau_1 = \tau; \tau_2 = 0$ (B) $\tau_1 = 0; \tau_2 = \tau$ (C) $\tau_1 = \tau_2 = \tau$. We need to get characteristics equations for all above stability analysis cases. We study the occurrence of any possible stability switching, resulting from the increase of the value of the time delays τ_1, τ_2 for the general characteristic equation $D(\tau_1, \tau_2)$. If we choose τ as a parameter, then the expression:

$$\begin{aligned} & D(\lambda, \tau) = P_n(\lambda, \tau) + Q_m(\lambda, \tau) e^{-\lambda \tau} \\ & n, m \in N_0; n > m \end{aligned} \quad (134)$$

$$\begin{aligned} & \det \begin{pmatrix} \Upsilon_{11} & \dots & \Upsilon_{15} \\ \vdots & \ddots & \vdots \\ \Upsilon_{51} & \cdots & \Upsilon_{55} \end{pmatrix} = (\sigma_1 - \lambda)(-\lambda) \\ & \det \begin{pmatrix} \sigma_5 - \lambda & 0 & 0 \\ 0 & -(\sigma_6 + \lambda) & \sigma_6 e^{-\lambda \tau_2} \\ 0 & \sigma_8 & (\sigma_9 e^{-\lambda \tau_2} - \lambda) \end{pmatrix} \\ & -\sigma_2 e^{-\lambda \tau_1} \det \begin{pmatrix} \sigma_5 - \lambda & 0 & 0 \\ 0 & -(\sigma_6 + \lambda) & \sigma_6 e^{-\lambda \tau_2} \\ 0 & \sigma_8 & (\sigma_9 e^{-\lambda \tau_2} - \lambda) \end{pmatrix} \\ & +\sigma_3 \{\det \begin{pmatrix} \sigma_5 e^{-\lambda \tau_1} & 0 & 0 \\ 0 & -(\sigma_6 + \lambda) & \sigma_6 e^{-\lambda \tau_2} \\ \sigma_7 e^{-\lambda \tau_1} & \sigma_8 & (\sigma_9 e^{-\lambda \tau_2} - \lambda) \end{pmatrix} \\ & +\lambda \det \begin{pmatrix} 0 & 0 & 0 \\ 0 & -(\sigma_6 + \lambda) & \sigma_6 e^{-\lambda \tau_2} \\ 0 & \sigma_8 & (\sigma_9 e^{-\lambda \tau_2} - \lambda) \end{pmatrix} \} \\ & +\sigma_4 e^{-\lambda \tau_2} \{\det \begin{pmatrix} \sigma_5 e^{-\lambda \tau_1} & \sigma_5 - \lambda & 0 \\ 0 & 0 & -(\sigma_6 + \lambda) \\ \sigma_7 e^{-\lambda \tau_1} & 0 & \sigma_8 \end{pmatrix} \\ & +\lambda \det \begin{pmatrix} 0 & \sigma_5 - \lambda & 0 \\ 0 & 0 & -(\sigma_6 + \lambda) \\ 0 & 0 & \sigma_8 \end{pmatrix} \} \end{aligned} \quad (135)$$

$$\begin{aligned} & \det \begin{pmatrix} 0 & 0 & 0 \\ 0 & -(\sigma_6 + \lambda) & \sigma_6 e^{-\lambda \tau_2} \\ 0 & \sigma_8 & (\sigma_9 e^{-\lambda \tau_2} - \lambda) \end{pmatrix} = 0 \\ & \det \begin{pmatrix} 0 & \sigma_5 - \lambda & 0 \\ 0 & 0 & -(\sigma_6 + \lambda) \\ 0 & 0 & \sigma_8 \end{pmatrix} = 0 \end{aligned} \quad (136)$$

We get the following expression:

$$\det\begin{pmatrix}\Upsilon_{11} & \dots & \Upsilon_{15}\\ \vdots & \ddots & \vdots\\ \Upsilon_{51} & \dots & \Upsilon_{55}\end{pmatrix} = (\sigma_1-\lambda)(-\lambda)$$
$$\det\begin{pmatrix}\sigma_5-\lambda & 0 & 0\\ 0 & -(\sigma_6+\lambda) & \sigma_6 e^{-\lambda\tau_2}\\ 0 & \sigma_8 & (\sigma_9 e^{-\lambda\tau_2}-\lambda)\end{pmatrix}$$
$$-\sigma_2 e^{-\lambda\tau_1}\det\begin{pmatrix}\sigma_5-\lambda & 0 & 0\\ 0 & -(\sigma_6+\lambda) & \sigma_6 e^{-\lambda\tau_2}\\ 0 & \sigma_8 & (\sigma_9 e^{-\lambda\tau_2}-\lambda)\end{pmatrix}$$
$$+\sigma_3\det\begin{pmatrix}\sigma_5 e^{-\lambda\tau_1} & 0 & 0\\ 0 & -(\sigma_6+\lambda) & \sigma_6 e^{-\lambda\tau_2}\\ \sigma_7 e^{-\lambda\tau_1} & \sigma_8 & (\sigma_9 e^{-\lambda\tau_2}-\lambda)\end{pmatrix}$$
$$+\sigma_4 e^{-\lambda\tau_2}\det\begin{pmatrix}\sigma_5 e^{-\lambda\tau_1} & \sigma_5-\lambda & 0\\ 0 & 0 & -(\sigma_6+\lambda)\\ \sigma_7 e^{-\lambda\tau_1} & 0 & \sigma_8\end{pmatrix} \tag{137}$$

First expression:

$$\det\begin{pmatrix}\sigma_5-\lambda & 0 & 0\\ 0 & -(\sigma_6+\lambda) & \sigma_6 e^{-\lambda\tau_2}\\ 0 & \sigma_8 & (\sigma_9 e^{-\lambda\tau_2}-\lambda)\end{pmatrix}$$
$$= (\sigma_5-\lambda)\det\begin{pmatrix}-(\sigma_6+\lambda) & \sigma_6 e^{-\lambda\tau_2}\\ \sigma_8 & (\sigma_9 e^{-\lambda\tau_2}-\lambda)\end{pmatrix}$$
$$= (\sigma_5-\lambda)\{-(\sigma_6+\lambda)(\sigma_9 e^{-\lambda\tau_2}-\lambda)-\sigma_8\sigma_6 e^{-\lambda\tau_2}\}$$
$$= (\sigma_5-\lambda)\{-\sigma_6\sigma_9 e^{-\lambda\tau_2}+\sigma_6\lambda-\lambda\sigma_9 e^{-\lambda\tau_2}$$
$$+\lambda^2-\sigma_8\sigma_6 e^{-\lambda\tau_2}\} = (\sigma_5-\lambda)\{\sigma_6\lambda+\lambda^2$$
$$-[\sigma_6\sigma_9+\sigma_8\sigma_6+\lambda\sigma_9]e^{-\lambda\tau_2}\} \tag{138}$$

$$\det\begin{pmatrix}\sigma_5-\lambda & 0 & 0\\ 0 & -(\sigma_6+\lambda) & \sigma_6 e^{-\lambda\tau_2}\\ 0 & \sigma_8 & (\sigma_9 e^{-\lambda\tau_2}-\lambda)\end{pmatrix}$$
$$= (\sigma_5-\lambda)\det\begin{pmatrix}-(\sigma_6+\lambda) & \sigma_6 e^{-\lambda\tau_2}\\ \sigma_8 & (\sigma_9 e^{-\lambda\tau_2}-\lambda)\end{pmatrix}$$
$$= (\sigma_5-\lambda)\{-(\sigma_6+\lambda)(\sigma_9 e^{-\lambda\tau_2}-\lambda)-\sigma_8\sigma_6 e^{-\lambda\tau_2}\}$$
$$= (\sigma_5-\lambda)\{-\sigma_6\sigma_9 e^{-\lambda\tau_2}+\sigma_6\lambda-\lambda\sigma_9 e^{-\lambda\tau_2}$$
$$+\lambda^2-\sigma_8\sigma_6 e^{-\lambda\tau_2}\}$$
$$= (\sigma_5-\lambda)\{\sigma_6\lambda+\lambda^2-[\sigma_6\sigma_9+\sigma_8\sigma_6+\lambda\sigma_9]e^{-\lambda\tau_2}\}$$
$$= \sigma_5\sigma_6\lambda+\sigma_5\lambda^2-\sigma_5[\sigma_6\sigma_9+\sigma_8\sigma_6+\lambda\sigma_9]e^{-\lambda\tau_2}$$
$$-\sigma_6\lambda^2-\lambda^3+\lambda[\sigma_6\sigma_9+\sigma_8\sigma_6+\lambda\sigma_9]e^{-\lambda\tau_2}$$
$$= \sigma_5\sigma_6\lambda+\sigma_5\lambda^2-[\sigma_5\sigma_6\sigma_9+\sigma_5\sigma_8\sigma_6$$
$$+\lambda\sigma_5\sigma_9]e^{-\lambda\tau_2}-\sigma_6\lambda^2-\lambda^3+[\lambda(\sigma_6\sigma_9+\sigma_8\sigma_6)$$
$$+\lambda^2\sigma_9]e^{-\lambda\tau_2} = \sigma_5\sigma_6\lambda+(\sigma_5-\sigma_6)\lambda^2-\lambda^3$$
$$+\{-\sigma_5\sigma_6(\sigma_9+\sigma_8)+\lambda(\sigma_6\sigma_9+\sigma_8\sigma_6$$
$$-\sigma_5\sigma_9)+\lambda^2\sigma_9\}e^{-\lambda\tau_2} \tag{139}$$

We define for simplicity:

$$\psi_1=\sigma_5\sigma_6;\psi_2=\sigma_5-\sigma_6;\psi_3=-\sigma_5\sigma_6(\sigma_9+\sigma_8) \tag{140}$$

$$\psi_4=\sigma_6\sigma_9+\sigma_8\sigma_6-\sigma_5\sigma_9 \tag{141}$$

Then we define

$$\det\begin{pmatrix}\sigma_5-\lambda & 0 & 0\\ 0 & -(\sigma_6+\lambda) & \sigma_6 e^{-\lambda\tau_2}\\ 0 & \sigma_8 & (\sigma_9 e^{-\lambda\tau_2}-\lambda)\end{pmatrix}$$
$$= \psi_1\lambda+\psi_2\lambda^2-\lambda^3+\{\psi_3+\lambda\psi_4+\lambda^2\sigma_9\}e^{-\lambda\tau_2} \tag{142}$$

Second expression:

$$\det\begin{pmatrix}\sigma_5 e^{-\lambda\tau_1} & 0 & 0\\ 0 & -(\sigma_6+\lambda) & \sigma_6 e^{-\lambda\tau_2}\\ \sigma_7 e^{-\lambda\tau_1} & \sigma_8 & (\sigma_9 e^{-\lambda\tau_2}-\lambda)\end{pmatrix} \tag{143}$$

$$\det\begin{pmatrix}\sigma_5 e^{-\lambda\tau_1} & 0 & 0\\ 0 & -(\sigma_6+\lambda) & \sigma_6 e^{-\lambda\tau_2}\\ \sigma_7 e^{-\lambda\tau_1} & \sigma_8 & (\sigma_9 e^{-\lambda\tau_2}-\lambda)\end{pmatrix}$$
$$= \sigma_5 e^{-\lambda\tau_1}\det\begin{pmatrix}-(\sigma_6+\lambda) & \sigma_6 e^{-\lambda\tau_2}\\ \sigma_8 & (\sigma_9 e^{-\lambda\tau_2}-\lambda)\end{pmatrix}$$
$$= \sigma_5 e^{-\lambda\tau_1}\{-(\sigma_6+\lambda)(\sigma_9 e^{-\lambda\tau_2}-\lambda)-\sigma_8\sigma_6 e^{-\lambda\tau_2}\}$$
$$= \sigma_5 e^{-\lambda\tau_1}\{-\sigma_6\sigma_9 e^{-\lambda\tau_2}+\sigma_6\lambda-\lambda\sigma_9 e^{-\lambda\tau_2}$$
$$+\lambda^2-\sigma_8\sigma_6 e^{-\lambda\tau_2}\}$$
$$= \sigma_5 e^{-\lambda\tau_1}\{\sigma_6\lambda+\lambda^2-[\sigma_6\sigma_9+\sigma_8\sigma_6+\lambda\sigma_9]e^{-\lambda\tau_2}\}$$
$$= (\sigma_6\lambda+\lambda^2)\sigma_5 e^{-\lambda\tau_1}-\sigma_5[\sigma_6\sigma_9+\sigma_8\sigma_6+\lambda\sigma_9]e^{-\lambda(\tau_2+\tau_1)}$$
$$\psi_5=\sigma_6\sigma_9+\sigma_8\sigma_6 \tag{144}$$

$$\det\begin{pmatrix}\sigma_5 e^{-\lambda\tau_1} & 0 & 0\\ 0 & -(\sigma_6+\lambda) & \sigma_6 e^{-\lambda\tau_2}\\ \sigma_7 e^{-\lambda\tau_1} & \sigma_8 & (\sigma_9 e^{-\lambda\tau_2}-\lambda)\end{pmatrix}$$
$$= (\sigma_6\lambda+\lambda^2)\sigma_5 e^{-\lambda\tau_1}-\sigma_5[\psi_5+\lambda\sigma_9]e^{-\lambda(\tau_2+\tau_1)} \tag{145}$$

Third expression:

$$\det\begin{pmatrix}\sigma_5 e^{-\lambda\tau_1} & (\sigma_5-\lambda) & 0\\ 0 & 0 & -(\sigma_6+\lambda)\\ \sigma_7 e^{-\lambda\tau_1} & 0 & \sigma_8\end{pmatrix}$$
$$= \sigma_5 e^{-\lambda\tau_1}\det\begin{pmatrix}0 & -(\sigma_6+\lambda)\\ 0 & \sigma_8\end{pmatrix}$$
$$-(\sigma_5-\lambda)\det\begin{pmatrix}0 & -(\sigma_6+\lambda)\\ \sigma_7 e^{-\lambda\tau_1} & \sigma_8\end{pmatrix}$$
$$= -(\sigma_5-\lambda)\det\begin{pmatrix}0 & -(\sigma_6+\lambda)\\ \sigma_7 e^{-\lambda\tau_1} & \sigma_8\end{pmatrix}$$
$$= -(\sigma_5-\lambda)\sigma_7 e^{-\lambda\tau_1}(\sigma_6+\lambda)$$
$$= -(\sigma_5-\lambda)\sigma_7(\sigma_6+\lambda)e^{-\lambda\tau_1}$$
$$= \sigma_7(-\sigma_5\sigma_6-\sigma_5\lambda+\lambda\sigma_6+\lambda^2)e^{-\lambda\tau_1}$$
$$= \sigma_7(-\sigma_5\sigma_6+\lambda[\sigma_6-\sigma_5]+\lambda^2)e^{-\lambda\tau_1} \tag{146}$$

$$\psi_1=\sigma_5\sigma_6;\psi_2=\sigma_5-\sigma_6 \Rightarrow -\psi_2=\sigma_6-\sigma_5 \tag{147}$$

$$\det\begin{pmatrix}\sigma_5 e^{-\lambda\tau_1} & (\sigma_5-\lambda) & 0\\ 0 & 0 & -(\sigma_6+\lambda)\\ \sigma_7 e^{-\lambda\tau_1} & 0 & \sigma_8\end{pmatrix}$$
$$= \sigma_7(-\psi_1-\lambda\psi_2+\lambda^2)e^{-\lambda\tau_1} \tag{148}$$

We integrate our expression in below $D(\tau_1,\tau_2)$ expression.

$$\det\begin{pmatrix}\Upsilon_{11} & \dots & \Upsilon_{15}\\ \vdots & \ddots & \vdots\\ \Upsilon_{51} & \dots & \Upsilon_{55}\end{pmatrix} = (\sigma_1-\lambda)(-\lambda)\det\begin{pmatrix}\sigma_5-\lambda & 0 & 0\\ 0 & -(\sigma_6+\lambda) & \sigma_6 e^{-\lambda\tau_2}\\ 0 & \sigma_8 & (\sigma_9 e^{-\lambda\tau_2}-\lambda)\end{pmatrix}$$
$$-\sigma_2 e^{-\lambda\tau_1}\det\begin{pmatrix}\sigma_5-\lambda & 0 & 0\\ 0 & -(\sigma_6+\lambda) & \sigma_6 e^{-\lambda\tau_2}\\ 0 & \sigma_8 & (\sigma_9 e^{-\lambda\tau_2}-\lambda)\end{pmatrix}$$
$$+\sigma_3\det\begin{pmatrix}\sigma_5 e^{-\lambda\tau_1} & 0 & 0\\ 0 & -(\sigma_6+\lambda) & \sigma_6 e^{-\lambda\tau_2}\\ \sigma_7 e^{-\lambda\tau_1} & \sigma_8 & (\sigma_9 e^{-\lambda\tau_2}-\lambda)\end{pmatrix}$$
$$+\sigma_4 e^{-\lambda\tau_2}\det\begin{pmatrix}\sigma_5 e^{-\lambda\tau_1} & \sigma_5-\lambda & 0\\ 0 & 0 & -(\sigma_6+\lambda)\\ \sigma_7 e^{-\lambda\tau_1} & 0 & \sigma_8\end{pmatrix} \quad (149)$$

$$\det\begin{pmatrix}\Upsilon_{11} & \dots & \Upsilon_{15}\\ \vdots & \ddots & \vdots\\ \Upsilon_{51} & \dots & \Upsilon_{55}\end{pmatrix} = (\sigma_1-\lambda)(-\lambda)[\psi_1\lambda+\psi_2\lambda^2$$
$$-\lambda^3+\{\psi_3+\lambda\psi_4+\lambda^2\sigma_9\}e^{-\lambda\tau_2}]-\sigma_2 e^{-\lambda\tau_1}[\psi_1\lambda$$
$$+\psi_2\lambda^2-\lambda^3+\{\psi_3+\lambda\psi_4+\lambda^2\sigma_9\}e^{-\lambda\tau_2}]$$
$$+\sigma_3[(\sigma_6\lambda+\lambda^2)\sigma_5 e^{-\lambda\tau_1}$$
$$-\sigma_5[\psi_5+\lambda\sigma_9]e^{-\lambda(\tau_2+\tau_1)}]$$
$$+\sigma_4 e^{-\lambda\tau_2}[\sigma_7(-\psi_1-\lambda\psi_2+\lambda^2)e^{-\lambda\tau_1}] \quad (150)$$

$$\det\begin{pmatrix}\Upsilon_{11} & \dots & \Upsilon_{15}\\ \vdots & \ddots & \vdots\\ \Upsilon_{51} & \dots & \Upsilon_{55}\end{pmatrix} = \psi_1\lambda^3+\psi_2\lambda^4-\lambda^5$$
$$+\{\psi_3\lambda^2+\lambda^3\psi_4+\lambda^4\sigma_9\}e^{-\lambda\tau_2}-\sigma_1\psi_1\lambda^2$$
$$-\sigma_1\psi_2\lambda^3+\sigma_1\lambda^4+\{-\sigma_1\psi_3\lambda-\sigma_1\psi_4\lambda^2 \quad (151)$$
$$-\sigma_1\sigma_9\lambda^3\}e^{-\lambda\tau_2}-(\psi_1\lambda+\psi_2\lambda^2-\lambda^3)\sigma_2 e^{-\lambda\tau_1}$$
$$-\sigma_2\{\psi_3+\lambda\psi_4+\lambda^2\sigma_9\}e^{-\lambda(\tau_1+\tau_2)}$$
$$+(\sigma_3\sigma_6\lambda+\sigma_3\lambda^2)\sigma_5 e^{-\lambda\tau_1}$$
$$-\sigma_3\sigma_5[\psi_5+\lambda\sigma_9]e^{-\lambda(\tau_2+\tau_1)}+(-\psi_1\sigma_4\sigma_7$$
$$-\lambda\psi_2\sigma_4\sigma_7+\lambda^2\sigma_4\sigma_7)e^{-\lambda(\tau_1+\tau_2)}$$

$$\det\begin{pmatrix}\Upsilon_{11} & \dots & \Upsilon_{15}\\ \vdots & \ddots & \vdots\\ \Upsilon_{51} & \dots & \Upsilon_{55}\end{pmatrix} = (\lambda^2-\sigma_1\lambda)[\psi_1\lambda+\psi_2\lambda^2$$
$$-\lambda^3+\{\psi_3+\lambda\psi_4+\lambda^2\sigma_9\}e^{-\lambda\tau_2}]$$
$$-[(\psi_1\lambda+\psi_2\lambda^2-\lambda^3)\sigma_2 e^{-\lambda\tau_1}$$
$$+\sigma_2\{\psi_3+\lambda\psi_4+\lambda^2\sigma_9\}e^{-\lambda(\tau_1+\tau_2)}]$$
$$+\sigma_3(\sigma_6\lambda+\lambda^2)\sigma_5 e^{-\lambda\tau_1}-\sigma_3\sigma_5[\psi_5+\lambda\sigma_9]e^{-\lambda(\tau_2+\tau_1)}$$
$$+(-\psi_1\sigma_4\sigma_7-\lambda\psi_2\sigma_4\sigma_7+\lambda^2\sigma_4\sigma_7)e^{-\lambda(\tau_1+\tau_2)} \quad (152)$$

$$\det\begin{pmatrix}\Upsilon_{11} & \dots & \Upsilon_{15}\\ \vdots & \ddots & \vdots\\ \Upsilon_{51} & \dots & \Upsilon_{55}\end{pmatrix} = -\sigma_1\psi_1\lambda^2$$
$$+(\psi_1-\sigma_1\psi_2)\lambda^3+(\psi_2+\sigma_1)\lambda^4-\lambda^5$$
$$-(\psi_1\lambda+\psi_2\lambda^2-\lambda^3)\sigma_2 e^{-\lambda\tau_1}$$
$$+(\sigma_3\sigma_6\lambda+\sigma_3\lambda^2)\sigma_5 e^{-\lambda\tau_1} \quad (153)$$
$$+\{\psi_3\lambda^2+\lambda^3\psi_4+\lambda^4\sigma_9\}e^{-\lambda\tau_2}$$
$$+\{-\sigma_1\psi_3\lambda-\sigma_1\psi_4\lambda^2-\sigma_1\sigma_9\lambda^3\}e^{-\lambda\tau_2}$$
$$-\sigma_2\{\psi_3+\lambda\psi_4+\lambda^2\sigma_9\}e^{-\lambda(\tau_1+\tau_2)}$$
$$-\sigma_3\sigma_5[\psi_5+\lambda\sigma_9]e^{-\lambda(\tau_2+\tau_1)}$$
$$+(-\psi_1\sigma_4\sigma_7-\lambda\psi_2\sigma_4\sigma_7+\lambda^2\sigma_4\sigma_7)e^{-\lambda(\tau_1+\tau_2)}$$

$$\det\begin{pmatrix}\Upsilon_{11} & \dots & \Upsilon_{15}\\ \vdots & \ddots & \vdots\\ \Upsilon_{51} & \dots & \Upsilon_{55}\end{pmatrix} = -\sigma_1\psi_1\lambda^2+(\psi_1-\sigma_1\psi_2)\lambda^3$$
$$+(\psi_2+\sigma_1)\lambda^4-\lambda^5+(-\psi_1\sigma_2\lambda-\psi_2\sigma_2\lambda^2+\sigma_2\lambda^3)e^{-\lambda\tau_1}$$
$$+(\sigma_3\sigma_6\sigma_5\lambda+\sigma_3\sigma_5\lambda^2)e^{-\lambda\tau_1}$$
$$+\{\psi_3\lambda^2+\lambda^3\psi_4+\lambda^4\sigma_9\}e^{-\lambda\tau_2}$$
$$+\{-\sigma_1\psi_3\lambda-\sigma_1\psi_4\lambda^2-\sigma_1\sigma_9\lambda^3\}e^{-\lambda\tau_2}$$
$$+\{-\sigma_2\psi_3-\lambda\sigma_2\psi_4-\lambda^2\sigma_2\sigma_9\}e^{-\lambda(\tau_1+\tau_2)}$$
$$+[-\sigma_3\sigma_5\psi_5-\lambda\sigma_3\sigma_5\sigma_9]e^{-\lambda(\tau_2+\tau_1)}$$
$$+(-\psi_1\sigma_4\sigma_7-\lambda\psi_2\sigma_4\sigma_7+\lambda^2\sigma_4\sigma_7)e^{-\lambda(\tau_1+\tau_2)} \quad (154)$$

$$\det\begin{pmatrix}\Upsilon_{11} & \dots & \Upsilon_{15}\\ \vdots & \ddots & \vdots\\ \Upsilon_{51} & \dots & \Upsilon_{55}\end{pmatrix} = -\sigma_1\psi_1\lambda^2+(\psi_1-\sigma_1\psi_2)\lambda^3$$
$$+(\psi_2+\sigma_1)\lambda^4-\lambda^5+\{(\sigma_3\sigma_6\sigma_5-\psi_1\sigma_2)\lambda$$
$$+(\sigma_3\sigma_5-\psi_2\sigma_2)\lambda^2+\sigma_2\lambda^3\}e^{-\lambda\tau_1}+\{-\sigma_1\psi_3\lambda$$
$$+(\psi_3-\sigma_1\psi_4)\lambda^2+(\psi_4-\sigma_1\sigma_9)\lambda^3+\lambda^4\sigma_9\}e^{-\lambda\tau_2}$$
$$+\{-\sigma_2\psi_3-\sigma_3\sigma_5\psi_5-\psi_1\sigma_4\sigma_7$$
$$-(\psi_2\sigma_4\sigma_7+\sigma_2\psi_4+\sigma_3\sigma_5\sigma_9)\lambda$$
$$+(\sigma_4\sigma_7-\sigma_2\sigma_9)\lambda^2\}e^{-\lambda(\tau_1+\tau_2)} \quad (155)$$

We define for simplicity the following parameters:

$$\theta_2=-\sigma_1\psi_1;\theta_3=\psi_1-\sigma_1\psi_2;\theta_4=\psi_2+\sigma_1;\theta_5=-1 \quad (156)$$

$$A_1=\sigma_3\sigma_6\sigma_5-\psi_1\sigma_2;A_2=\sigma_3\sigma_5-\psi_2\sigma_2;A_3=\sigma_2 \quad (157)$$

$$B_1=-\sigma_1\psi_3;B_2=\psi_3-\sigma_1\psi_4$$
$$B_3=\psi_4-\sigma_1\sigma_9;B_4=\sigma_9 \quad (158)$$

$$C_0=-\sigma_2\psi_3-\sigma_3\sigma_5\psi_5-\psi_1\sigma_4\sigma_7$$
$$C_1=-(\psi_2\sigma_4\sigma_7+\sigma_2\psi_4+\sigma_3\sigma_5\sigma_9) \quad (159)$$

$$C_2=\sigma_4\sigma_7-\sigma_2\sigma_9 \quad (160)$$

$$\det\begin{pmatrix} \Upsilon_{11} & \cdots & \Upsilon_{15} \\ \vdots & \ddots & \vdots \\ \Upsilon_{51} & \cdots & \Upsilon_{55} \end{pmatrix} = \sum_{l=2}^{5}\Theta_l\lambda^l + [\sum_{k=1}^{3}A_k\lambda^k]e^{-\lambda\tau_1} + [\sum_{k=1}^{4}B_k\lambda^k]e^{-\lambda\tau_2} + [\sum_{k=0}^{2}C_k\lambda^k]e^{-\lambda(\tau_1+\tau_2)} \quad (161)$$

$$D(\tau_1,\tau_2) = \sum_{l=2}^{5}\Theta_l\lambda^l + [\sum_{k=1}^{3}A_k\lambda^k]e^{-\lambda\tau_1} + [\sum_{k=1}^{4}B_k\lambda^k]e^{-\lambda\tau_2} + [\sum_{k=0}^{2}C_k\lambda^k]e^{-\lambda(\tau_1+\tau_2)} \quad (162)$$

Three cases:

$$(A)\,\tau_1=\tau;\tau_2=0\,(B)\,\tau_1=0;\tau_2=\tau \qquad (C)\,\tau_1=\tau_2=\tau \quad (163)$$

IV. RFID TAG RECEIVER DETECTOR CHARACTERISTIC EQUATION AND STABILITY SWITCHING $\tau_1=\tau;\tau_2=0$

We get and analyze the characteristic equation of RFID TAG receiver for $\tau_1=\tau;\tau_2=0$.

$$\tau_1=\tau;\tau_2=0; D(\tau)=\sum_{l=2}^{5}\Theta_l\lambda^l+[\sum_{k=1}^{4}B_k\lambda^k] + [\sum_{k=1}^{3}A_k\lambda^k]e^{-\lambda\tau}+[\sum_{k=0}^{2}C_k\lambda^k]e^{-\lambda\tau} \quad (164)$$

$$D(\tau_1=\tau;\tau_2=0)=\sum_{l=2}^{5}\Theta_l\lambda^l+[\sum_{k=1}^{4}B_k\lambda^k] + [\sum_{k=1}^{3}A_k\lambda^k]e^{-\lambda\tau}+[\sum_{k=0}^{2}C_k\lambda^k]e^{-\lambda\tau}$$
$$D(\tau_1=\tau;\tau_2=0)=B_1\lambda+\sum_{l=2}^{4}(\Theta_l+B_l)\lambda^l + \Theta_5\lambda^5+[C_0+\sum_{l=1}^{2}(A_l+C_l)\lambda^l+A_3\lambda^3]e^{-\lambda\tau} \quad (165)$$

$$D(\lambda,\tau)=P_n(\lambda,\tau)+Q_m(\lambda,\tau)e^{-\lambda\tau} \qquad n,m\in N_0; n>m \quad (166)$$

$$P_n(\lambda,\tau)=B_1\lambda+\sum_{l=2}^{4}(\Theta_l+B_l)\lambda^l + \Theta_5\lambda^5; n=5$$
$$Q_m(\lambda,\tau)=[C_0+\sum_{l=1}^{2}(A_l+C_l)\lambda^l + A_3\lambda^3]; m=3 \quad (167)$$

$$P_n(\lambda,\tau)=\sum_{k=0}^{n}P_k(\tau)\,\lambda^k=P_0(\tau)+P_1(\tau)\lambda + P_2(\tau)\,\lambda^2+P_3(\tau)\,\lambda^3+...$$
$$Q_m(\lambda,\tau)=\sum_{k=0}^{m}q_k(\tau)\,\lambda^k=q_0(\tau)+q_1(\tau)\lambda + q_2(\tau)\,\lambda^2+........ \quad (168)$$

$$D(\lambda,\tau)=P_n(\lambda,\tau)+Q_m(\lambda,\tau)e^{-\lambda\tau} \qquad n=5;m=3;n>m \quad (169)$$

$$P_n(\lambda,\tau)=\sum_{k=0}^{n}P_k(\tau)\,\lambda^k=P_0(\tau)+P_1(\tau)\lambda + P_2(\tau)\lambda^2+P_3(\tau)\lambda^3+P_4(\tau)\lambda^4+P_5(\tau)\lambda^5 \quad (170)$$

$$P_0=0;P_1=B_1;P_2=\Theta_2+B_2;P_3=\Theta_3+B_3 \qquad P_4=\Theta_4+B_4;P_5=\Theta_5 \quad (171)$$

$$Q_m(\lambda,\tau)=\sum_{k=0}^{m}q_k(\tau)\lambda^k=q_0(\tau) + q_1(\tau)\lambda+q_2(\tau)\lambda^2+q_3(\tau)\lambda^3;\ q_0(\tau)=C_0 \qquad q_1(\tau)=A_1+C_1;\ q_2(\tau)=A_2+C_2;q_3(\tau)=A_3 \quad (172)$$

The homogeneous system for $X,Y,I_{L_1},I_{R_j},I_{R_S}$ leads to a characteristic equation for the eigenvalue λ having the form

$$P(\lambda,\tau)+Q(\lambda,\tau)e^{-\lambda\tau}=0 \qquad P(\lambda)=\sum_{j=0}^{5}a_j\lambda^j;Q(\lambda)=\sum_{j=0}^{3}c_j\lambda^j \quad (173)$$

The coefficients $\{a_j(q_i,q_k,\tau),c_j(q_i,q_k,\tau)\}\in R$ depend on q_i,q_k and delay τ. q_i,q_k are any Schottky detector's global parameters, other parameters kept as a constant.

$$a_0=0;a_1=B_1;a_2=\Theta_2+B_2;a_3=\Theta_3+B_3 \quad (174)$$

$$a_4=\Theta_4+B_4;a_5=\Theta_5;\ c_0(\tau)=C_0 \qquad c_1(\tau)=A_1+C_1;\ c_2(\tau)=A_2+C_2;\ c_3(\tau)=A_3 \quad (175)$$

Unless strictly necessary, the designation of the varied arguments (q_i,q_k) will subsequently be omitted from P, Q, a_j, and c_j. The coefficients a_j, c_j are continuous, and differentiable functions of their arguments, and direct substitution shows that $a_0+c_0\neq0$ for $q_i,q_k\in R_+$; that is, λ=0 is not of $P(\lambda)+Q(\lambda)e^{-\lambda\tau}=0$ [7] [8]. Furthermore, P(λ), Q(λ) are analytic functions of λ, for which the following requirements of the analysis (Kuang J and Cong Y 2005 ; Kuang Y 1993) can also be verified in the present case:

.

(a) If $\lambda=i\omega,\omega\in R$, then $P(i\omega)+Q(i\omega)\neq0$
(b) If $|Q(\lambda)/P(\lambda)|$ is bounded for $|\lambda|\to\infty;\mathrm{Re}\lambda\geq0$. No roots bifurcation from ∞.
(c) $F(\omega)=|P(i\omega)|^2-|Q(i\omega)|^2$ has a finite number of zeros. Indeed, this is a polynomial in ω.
(d) Each positive root $\omega(q_i,q_k)$ of F(ω)=0 is continuous and differentiable with respect to q_i,q_k.

.

We assume that $P_n(\lambda,\tau)$ and $Q_m(\lambda,\tau)$ cannot have common imaginary roots. That is for any real number ω:

$$P_n(\lambda=i\omega,\tau)+Q_m(\lambda=i\omega,\tau)\neq0 \quad (176)$$

$$p_n(\lambda=i\omega,\tau)=B_1 i\omega+\sum_{l=2}^{4}(\Theta_l+B_l)(i\omega)^l + \Theta_5(i\omega)^5=i\omega B_1+\sum_{l=2}^{4}(\Theta_l+B_l)i^l\omega^l+i\Theta_5\omega^5 \quad (177)$$

$$\sum_{l=2}^{4}(\Theta_l+\mathrm{B}_l)i^l\omega^l=-(\Theta_2+\mathrm{B}_2)\omega^2 +(\Theta_2+\mathrm{B}_2)\omega^4-(\Theta_2+\mathrm{B}_2)\omega^3 i \quad (178)$$

$$p_n(\lambda=i\omega,\tau)=-(\Theta_2+\mathrm{B}_2)\omega^2+(\Theta_2+\mathrm{B}_2)\omega^4 +i[\omega\mathrm{B}_1-(\Theta_2+\mathrm{B}_2)\omega^3+\Theta_5\omega^5] \quad (179)$$

$$Q_m(\lambda=i\omega,\tau)=C_0 +\sum_{l=1}^{2}(\mathrm{A}_l+C_l)(i\omega)^l-i\mathrm{A}_3\omega^3$$
$$\sum_{l=1}^{2}(\mathrm{A}_l+C_l)(i\omega)^l=i\omega(\mathrm{A}_1+C_1) -(\mathrm{A}_2+C_2)\omega^2 \quad (180)$$

$$Q_m(\lambda=i\omega,\tau)=C_0-(A_2+C_2)\omega^2 +i[\omega(A_1+C_1)-A_3\omega^3] \quad (181)$$

$$p_n(\lambda=i\omega,\tau)+Q_m(\lambda=i\omega,\tau) =C_0-(\Theta_2+\mathrm{B}_2)\omega^2-(\mathrm{A}_2+C_2)\omega^2 +(\Theta_2+\mathrm{B}_2)\omega^4+i[\omega\mathrm{B}_1+\omega(\mathrm{A}_1+C_1) -(\Theta_2+\mathrm{B}_2)\omega^3-\mathrm{A}_3\omega^3+\Theta_5\omega^5]\neq 0 \quad (182)$$

$$p_n(\lambda=i\omega,\tau)+Q_m(\lambda=i\omega,\tau) =C_0-(\Theta_2+\mathrm{B}_2+\mathrm{A}_2+C_2)\omega^2 +(\Theta_2+\mathrm{B}_2)\omega^4+i[\omega(\mathrm{A}_1+C_1+\mathrm{B}_1) -(\Theta_2+\mathrm{B}_2+\mathrm{A}_3)\omega^3+\Theta_5\omega^5]\neq 0 \quad (183)$$

$$|P(i\omega,\tau)|^2=[-(\Theta_2+\mathrm{B}_2)\omega^2+(\Theta_2+\mathrm{B}_2)\omega^4]^2 +[\omega\mathrm{B}_1-(\Theta_2+\mathrm{B}_2)\omega^3+\Theta_5\omega^5]^2=(\Theta_2+\mathrm{B}_2)^2\omega^4 +(\Theta_2+\mathrm{B}_2)^2\omega^8-2(\Theta_2+\mathrm{B}_2)^2\omega^6+\omega^2\mathrm{B}_1^2 -\mathrm{B}_1(\Theta_2+\mathrm{B}_2)\omega^4+\mathrm{B}_1\Theta_5\omega^6-(\Theta_2+\mathrm{B}_2)\mathrm{B}_1\omega^4 +(\Theta_2+\mathrm{B}_2)^2\omega^6-(\Theta_2+\mathrm{B}_2)\Theta_5\omega^8+\Theta_5\mathrm{B}_1\omega^6 -\Theta_5(\Theta_2+\mathrm{B}_2)\omega^8+\Theta_5^2\omega^{10} \quad (184)$$

$$|P(i\omega,\tau)|^2=\omega^2\mathrm{B}_1^2 +[(\Theta_2+\mathrm{B}_2)-2\mathrm{B}_1](\Theta_2+\mathrm{B}_2)\omega^4 +[2\mathrm{B}_1\Theta_5-(\Theta_2+\mathrm{B}_2)^2]\omega^6+[(\Theta_2+\mathrm{B}_2) -2\Theta_5](\Theta_2+\mathrm{B}_2)\omega^8+\Theta_5^2\omega^{10} \quad (185)$$

$$|Q(i\omega,\tau)|^2=[C_0-(\mathrm{A}_2+C_2)\omega^2]^2 +[\omega(\mathrm{A}_1+C_1)-\mathrm{A}_3\omega^3]^2=C_0^2+(\mathrm{A}_2+C_2)^2\omega^4 -2C_0(\mathrm{A}_2+C_2)\omega^2+\omega^2(\mathrm{A}_1+C_1)^2+\mathrm{A}_3^2\omega^6 -2(\mathrm{A}_1+C_1)\mathrm{A}_3\omega^4 \quad (186)$$

$$|Q(i\omega,\tau)|^2=C_0^2+[(\mathrm{A}_1+C_1)^2 -2C_0(\mathrm{A}_2+C_2)]\omega^2+[(\mathrm{A}_2+C_2)^2 -2(\mathrm{A}_1+C_1)\mathrm{A}_3]\omega^4+\mathrm{A}_3^2\omega^6 \quad (187)$$

$$F(\omega,\tau)=|P(i\omega,\tau)|^2-|Q(i\omega,\tau)|^2=\omega^2\mathrm{B}_1^2 +[(\Theta_2+\mathrm{B}_2)-2\mathrm{B}_1](\Theta_2+\mathrm{B}_2)\omega^4 +[2\mathrm{B}_1\Theta_5-(\Theta_2+\mathrm{B}_2)^2]\omega^6+[(\Theta_2+\mathrm{B}_2) -2\Theta_5](\Theta_2+\mathrm{B}_2)\omega^8+\Theta_5^2\omega^{10} -\{C_0^2+[(\mathrm{A}_1+C_1)^2-2C_0(\mathrm{A}_2+C_2)]\omega^2 +[(\mathrm{A}_2+C_2)^2-2(\mathrm{A}_1+C_1)\mathrm{A}_3]\omega^4+\mathrm{A}_3^2\omega^6\} \quad (188)$$

$$F(\omega,\tau)=|P(i\omega,\tau)|^2-|Q(i\omega,\tau)|^2=-C_0^2 +\{\mathrm{B}_1^2-[(\mathrm{A}_1+C_1)^2-2C_0(\mathrm{A}_2+C_2)]\}\omega^2 +\{[(\Theta_2+\mathrm{B}_2)-2\mathrm{B}_1](\Theta_2+\mathrm{B}_2)-[(\mathrm{A}_2+C_2)^2 -2(\mathrm{A}_1+C_1)\mathrm{A}_3]\}\omega^4+\{[2\mathrm{B}_1\Theta_5-(\Theta_2+\mathrm{B}_2)^2] -\mathrm{A}_3^2\}\omega^6+[(\Theta_2+\mathrm{B}_2)-2\Theta_5](\Theta_2+\mathrm{B}_2)\omega^8 +\Theta_5^2\omega^{10} \quad (189)$$

We define the following parameters for simplicity: $\Pi_0,\Pi_2,\Pi_4,\Pi_6,\Pi_8,\Pi_{10}$.

$$\Pi_0=-C_0^2;\Pi_2=\mathrm{B}_1^2-[(\mathrm{A}_1+C_1)^2 -2C_0(\mathrm{A}_2+C_2)]$$
$$\Pi_4=[(\Theta_2+\mathrm{B}_2)-2\mathrm{B}_1](\Theta_2+\mathrm{B}_2) -[(\mathrm{A}_2+C_2)^2-2(\mathrm{A}_1+C_1)\mathrm{A}_3]$$
$$\Pi_6=[2\mathrm{B}_1\Theta_5-(\Theta_2+\mathrm{B}_2)^2]-\mathrm{A}_3^2$$
$$\Pi_8=[(\Theta_2+\mathrm{B}_2)-2\Theta_5](\Theta_2+\mathrm{B}_2);\Pi_{10}=\Theta_5^2 \quad (190)$$

Hence $F(\omega,\tau)=0$ implies $\sum_{k=0}^{5}\prod_{2k}\omega^{2k}=0$ and its roots are given by solving the above polynomial.

$$P_R(i\omega,\tau)=-(\Theta_2+B_2)\omega^2+(\Theta_2+B_2)\omega^4 \quad (191)$$

$$P_I(i\omega,\tau)=\omega\mathrm{B}_1-(\Theta_2+\mathrm{B}_2)\omega^3+\Theta_5\omega^5$$
$$Q_R(i\omega,\tau)=C_0-(\mathrm{A}_2+C_2)\omega^2 \quad (192)$$

$$Q_I(i\omega,\tau)=\omega(A_1+C_1)-A_3\omega^3 \quad (193)$$

$$\sin\theta(\tau)=\frac{-P_R(i\omega,\tau)Q_I(i\omega,\tau)+P_I(i\omega,\tau)Q_R(i\omega,\tau)}{|Q(i\omega,\tau)|^2} \quad (194)$$

$$\cos\theta(\tau)=-\frac{P_R(i\omega,\tau)Q_R(i\omega,\tau)+P_I(i\omega,\tau)Q_I(i\omega,\tau)}{|Q(i\omega,\tau)|^2} \quad (195)$$

We use different parameters terminology from our last characteristics parameters definition:

$$k\to j;p_k(\tau)\to a_j;q_k(\tau)\to c_j$$
$$n=5;m=3;n>m \quad (196)$$

$$P_n(\lambda,\tau)\to P(\lambda);Q_m(\lambda,\tau)\to Q(\lambda) \quad (197)$$

$$P(\lambda)=\sum_{j=0}^{5}a_j\lambda^j;Q(\lambda)=\sum_{j=0}^{3}c_j\lambda^j \quad (198)$$

$$P_\lambda=a_0+a_1\lambda+a_2\lambda^2+a_3\lambda^3+a_4\lambda^4+a_5\lambda^5$$
$$Q_\lambda=c_0+c_1\lambda+c_2\lambda^2+c_3\lambda^3 \quad (199)$$

$n,m\in N_0,n>m$ and $a_j,c_j:R_{+0}\to R$ are continuous and differentiable function of τ such that $a_0+c_0\neq 0$. In the following "—" denotes complex and conjugate. $P(\lambda),Q(\lambda)$ are analytic functions in λ and differentiable in τ. The coefficients $a_j(L_P,L_1,C_f,R_{in},R_S,C_P,R_j,\tau,...)\in R$ and $c_j(L_P,L_1,C_f,R_{in},R_S,C_P,R_j,\tau,...)\in R$ depend on RFID TAG detector system's $L_P,L_1,C_f,R_{in},R_S,C_P,R_j,\tau,...$

values. Unless strictly necessary, the designation of the varied arguments: $(L_P, L_1, C_f, R_{in}, R_S, C_P, R_j, \tau, ...)$ will subsequently be omitted from P, Q, a_j, c_j. The coefficients a_j, c_j are continuous, and differentiable functions of their arguments, and direct substitution shows that $a_0 + c_0 \neq 0$.

$$a_0 = 0; c_0(\tau) = c_0 \tag{200}$$

$$C_0 = -\sigma_2\psi_3 - \sigma_3\sigma_5\psi_5 - \psi_1\sigma_4\sigma_7 \tag{201}$$

$$-\sigma_2\psi_3 - \sigma_3\sigma_5\psi_5 - \psi_1\sigma_4\sigma_7 \neq 0 \tag{202}$$

$$\begin{gathered}-[\tfrac{R_{in}^2}{L_1 L_p} - \tfrac{1}{C_1 L_p} - \tfrac{1}{C_p L_p}]\psi_3 + \tfrac{R_{in}^2}{L_1 L_p}\tfrac{R_{in}}{L_1}\psi_5 \\ -\psi_1\tfrac{1}{C_p L_p}\tfrac{1}{R_s C_p} \neq 0\end{gathered} \tag{203}$$

$\forall L_P, L_1, C_f, R_{in}, R_S, C_P, R_j, \tau, ... \in R_+$ i.e $\lambda = 0$ is not a root of the characteristic equation. Furthermore $P(\lambda), Q(\lambda)$ are analytic functions of λ for which the following requirements of the analysis (see Kuang, 1993, section 3.4) can also be verified in the present case [6] [7].

.

(a) If $\lambda = i\omega; \omega \in R$ then $P(i\omega) + Q(i\omega) \neq 0$, i.e P and Q have no common imaginary roots. This condition was verified numerically in the entire $(L_P, L_1, C_f, R_{in}, R_S, C_P, R_j, \tau, ...)$ domain of interest.
(b) $|P(\lambda)/Q(\lambda)|$ is bounded for $|\lambda| \to \infty; \mathrm{Re}\lambda \geq 0$. No roots bifurcation from ∞. Indeed, in the limit:

$$|\frac{Q(\lambda)}{P(\lambda)}| = |\frac{c_0 + c_1\lambda + c_2\lambda^2 + c_3\lambda^3}{a_0 + a_1\lambda + a_2\lambda^2 + a_3\lambda^3 + a_4\lambda^4 + a_5\lambda^5}| \tag{204}$$

(c) The following expressions exist:

$$F(\omega) = |P(i\omega)|^2 - |Q(i\omega)|^2 \tag{205}$$

$$F(\omega,\tau) = |P(i\omega,\tau)|^2 - |Q(i\omega,\tau)|^2 = \sum_{k=0}^{5}\Pi_{2k}\omega^{2k} \tag{206}$$

Has at most a finite number of zeros. Indeed, this is a polynomial in ω(Degree in ω^{10}).
(d) Each positive root $\omega(L_P, L_1, C_f, R_{in}, R_S, C_P, R_j, \tau, ...)$ of $F(\omega) = 0$ is continuous and differentiable with respect to $L_P, L_1, C_f, R_{in}, R_S, C_P, R_j, \tau, ...$. The condition can only be assessed numerically.

.

In addition, since the coefficients in P and Q are real, we have $\overline{P(-i\omega)} = P(i\omega)$, and $\overline{Q(-i\omega)} = Q(i\omega)$ thus, $\omega > 0$ maybe on eigenvalue of characteristic equations. The analysis consists in identifying the roots of the characteristic equation situated on the imaginary axis of the complex λ – plane, whereby increasing the parameters: $L_P, L_1, C_f, R_{in}, R_S, C_P, R_j, \tau, ...$ Re λ may, at the crossing, change its sign from (-) to (+), i.e. from a stable focus

$$\begin{gathered}E^{(0)}(X^{(0)}, Y^{(0)}, I_{L_1}^{(0)}, I_{R_j}^{(0)}, I_{R_S}^{(0)}) \\ |_{V(t)|_{A_0 \gg |f(t)=A_0+f(t)\approx A_0}} = (0,0,0,0,0) \\ \tfrac{dV(t)}{dt}|_{A_0 \gg |f(t)| = \frac{df(t)}{dt} \to \varepsilon}\end{gathered} \tag{207}$$

to an unstable one, or vice versa. This feature may be further assessed by examining the sign of the partial derivatives with respect to $L_P, L_1, C_f, R_{in}, R_S, C_P, R_j, \tau, ...$ and system parameters.

$$\begin{gathered}\wedge^{-1}(L_P) = (\tfrac{\partial \mathrm{Re}\lambda}{\partial L_P})_{\lambda=i\omega} \\ L_1, C_f, R_{in}, R_s, C_P, R_j, \tau, ... = const\end{gathered} \tag{208}$$

$$\begin{gathered}\wedge^{-1}(L_1) = (\tfrac{\partial \mathrm{Re}\lambda}{\partial L_1})_{\lambda=i\omega} \\ L_P, C_f, R_{in}, R_s, C_P, R_j, \tau, ... = const\end{gathered} \tag{209}$$

$$\begin{gathered}\wedge^{-1}(C_f) = (\tfrac{\partial \mathrm{Re}\lambda}{\partial C_f})_{\lambda=i\omega} \\ L_P, L_1, R_{in}, R_s, C_P, R_j, \tau, ... = const\end{gathered} \tag{210}$$

$$\begin{gathered}\wedge^{-1}(R_{in}) = (\tfrac{\partial \mathrm{Re}\lambda}{\partial R_{in}})_{\lambda=i\omega} \\ L_P, L_1, C_f, R_s, C_P, R_j, \tau, ... = const\end{gathered} \tag{211}$$

$$\begin{gathered}\wedge^{-1}(R_{in}) = (\tfrac{\partial \mathrm{Re}\lambda}{\partial R_{in}})_{\lambda=i\omega} \\ L_P, L_1, C_f, R_s, C_P, R_j, \tau, ... = const\end{gathered} \tag{212}$$

$$\begin{gathered}\wedge^{-1}(R_s) = (\tfrac{\partial \mathrm{Re}\lambda}{\partial R_s})_{\lambda=i\omega} \\ L_P, L_1, C_f, R_{in}, C_P, R_j, \tau, ... = const\end{gathered} \tag{213}$$

$$\begin{gathered}\wedge^{-1}(C_P) = (\tfrac{\partial \mathrm{Re}\lambda}{\partial C_P})_{\lambda=i\omega} \\ L_P, L_1, C_f, R_{in}, R_s, R_j, \tau, ... = const\end{gathered} \tag{214}$$

$$\begin{gathered}\wedge^{-1}(R_j) = (\tfrac{\partial \mathrm{Re}\lambda}{\partial R_j})_{\lambda=i\omega} \\ L_P, L_1, C_f, R_{in}, R_s, C_P, \tau, ... = const\end{gathered} \tag{215}$$

$$\begin{gathered}\wedge^{-1}(\tau) = (\tfrac{\partial \mathrm{Re}\lambda}{\partial R_j})_{\lambda=i\omega} \\ L_P, L_1, C_f, R_{in}, R_s, R_j, C_P, ... = const\end{gathered} \tag{216}$$

$$P(\lambda) = P_R(\lambda) + iP_I(\lambda); Q(\lambda) = Q_R(\lambda) + iQ_I(\lambda) \tag{217}$$

When writing and inserting $\lambda = i\omega$ into active RFID Schottky detector system's characteristic equation ω must satisfy the following equations.

$$\sin\omega\tau = g(\omega) = \frac{-P_R(i\omega)Q_I(i\omega) + P_I(i\omega)Q_R(i\omega)}{|Q(i\omega)|^2} \tag{218}$$

$$\cos\omega\tau = h(\omega) = -\frac{P_R(i\omega)Q_R(i\omega) + P_I(i\omega)Q_I(i\omega)}{|Q(i\omega)|^2} \tag{219}$$

Where $|Q(i\omega)|^2 \neq 0$ in view of requirement (a) above, and $(g, h) \in R$. Furthermore, it follows above $\sin\omega\tau$ and $\cos\omega\tau$ equations that, by squaring and adding the sides, ω must be a positive root of $F(\omega) = |P(i\omega)|^2 - |Q(i\omega)|^2 = 0$. Note: $F(\omega)$ is dependent on τ. Now it is important to notice that if $\tau \notin I$ (assume that $I \subseteq R_{+0}$ is the set where $\omega(\tau)$ is a positive root of $F(\omega)$ and for, $\tau \notin I$, $\omega(\tau)$ is not defined. Then for all τ in I, $\omega(\tau)$ is satisfied that $F(\omega,\tau) = 0$. Then there are no positive $\omega(\tau)$ solutions for $F(\omega,\tau) = 0$, and we cannot have stability switches. For $\tau \in I$ where $\omega(\tau)$ is a positive solution of $F(\omega,\tau) = 0$, we can define the angle $\theta(\tau) \in [0, 2\pi]$ as the solution of $\sin\theta(\tau) = ...; \cos\theta(\tau) = ...$

$$\sin\theta(\tau) = \frac{-P_R(i\omega)Q_I(i\omega) + P_I(i\omega)Q_R(i\omega)}{|Q(i\omega)|^2} \tag{220}$$

$$\cos\theta(\tau) = -\frac{P_R(i\omega)Q_R(i\omega) + P_I(i\omega)Q_I(i\omega)}{|Q(i\omega)|^2} \quad (221)$$

And the relation between the argument $\theta(\tau)$ and $\tau\omega(\tau)$ for $\tau \in I$ must be as describe below.

$$\omega(\tau)\tau = \theta(\tau) + 2n\pi \;\forall\; n \in N_0 \quad (222)$$

Hence we can define the maps $\tau_n : I \to R_{+0}$ given by

$$\tau_n(\tau) = \frac{\theta(\tau) + 2n\pi}{\omega(\tau)}; n \in N_0, \tau \in I \quad (223)$$

Let us introduce the functions:

$$I \to R; S_n(\tau) = \tau - \tau_n(\tau),\; \tau \in I,\; n \in N_0 \quad (224)$$

that is continuous and differentiable in τ. In the following, the subscripts $\lambda, \omega, L_P, L_1, C_f, R_{in}, R_S, C_P, R_j, ...$ indicate the corresponding partial derivatives. Let us first concentrate on $\Lambda(x)$, remember in $\lambda(L_P, L_1, C_f, R_{in}, R_S, C_P, R_j, ...)$ and $\omega(L_P, L_1, C_f, R_{in}, R_S, C_P, R_j, ...)$, and keeping all parameters except one (x) and τ. The derivation closely follows that in reference [BK]. Differentiating RFID TAG detector system characteristic equation $P(\lambda) + Q(\lambda)e^{-\lambda\tau} = 0$ with respect to specific parameter (x), and inverting the derivative, for convenience, one calculates: $x = L_P, L_1, C_f, R_{in}, R_S, C_P, R_j, ...$

$$(\frac{\partial\lambda}{\partial x})^{-1} = \frac{\begin{array}{c}-P_\lambda(\lambda,x)Q(\lambda,x) + Q_\lambda(\lambda,x)P(\lambda,x)\\ -\tau P(\lambda,x)Q(\lambda,x)\end{array}}{P_x(\lambda,x)Q(\lambda,x) - Q_x(\lambda,x)P(\lambda,x)} \quad (225)$$

Where $P_\lambda = \frac{\partial P}{\partial \lambda}, ...$ etc., substituting $\lambda = i\omega$ and bearing

$$\overline{P(-i\omega)} = P(i\omega); \overline{Q(-i\omega)} = Q(i\omega) \quad (226)$$

$$iP_\lambda(i\omega) = P_\omega(i\omega); iQ_\lambda(i\omega) = Q_\omega(i\omega) \quad (227)$$

and that on the surface $|P(i\omega)|^2 = |Q(i\omega)|^2$, one obtains:

$$\begin{array}{l}(\frac{\partial\lambda}{\partial x})^{-1}|_{\lambda=i\omega}\\ = (\frac{\begin{array}{c}iP_\omega(i\omega,x)\overline{P(i\omega,x)}\\ +iQ_\lambda(i\omega,x)\overline{Q(\lambda,x)} - \tau|P(i\omega,x)|^2\end{array}}{P_x(i\omega,x)\overline{P(i\omega,x)} - Q_x(i\omega,x)\overline{Q(i\omega,x)}})\end{array} \quad (228)$$

Upon separating into real and imaginary parts, with

$$P = P_R + iP_I; Q = Q_R + iQ_I \quad (229)$$

$$\begin{array}{l}P_\omega = P_{R\omega} + iP_{I\omega}; Q_\omega = Q_{R\omega} + iQ_{I\omega}\\ P_x = P_{Rx} + iP_{Ix}\end{array} \quad (230)$$

$$Q_x = Q_{Rx} + iQ_{Ix}; P^2 = P_R^2 + P_I^2 \quad (231)$$

When (x) can be any RFID Schottky detector parameter's $L_P, L_1, C_f, R_{in}, ...$ and time delay τ etc. Where for convenience, we have dropped the arguments $(i\omega, x)$, and where

$$F_\omega = 2[(P_{R\omega}P_R + P_{I\omega}P_I) - (Q_{R\omega}Q_R + Q_{I\omega}Q_I)] \quad (232)$$

$$F_x = 2[(P_{Rx}P_R + P_{Ix}P_I) - (Q_{Rx}Q_R + Q_{Ix}Q_I)] \quad (233)$$

$\omega_x = {-F_x}/{F_\omega}$. We define U and V:

$$\begin{array}{l}U = (P_R P_{I\omega} - P_I P_{R\omega}) - (Q_R Q_{I\omega} - Q_I Q_{R\omega})\\ V = (P_R P_{Ix} - P_I P_{Rx}) - (Q_R Q_{Ix} - Q_I Q_{Rx})\end{array} \quad (234)$$

We choose our specific parameter as time delay $x = \tau$.

$$Q_I = \omega(A_1 + C_1) - A_3\omega^3 \quad (235)$$

$$\begin{array}{l}P_R = -(\Theta_2 + \mathrm{B}_2)\omega^2 + (\Theta_2 + \mathrm{B}_2)\omega^4\\ P_I = \omega\mathrm{B}_1 - (\Theta_2 + \mathrm{B}_2)\omega^3 + \Theta_5\omega^5\\ Q_R = C_0 - (\mathrm{A}_2 + C_2)\omega^2\end{array} \quad (236)$$

$$\begin{array}{l}P_{R\omega} = 4(\Theta_2 + \mathrm{B}_2)\omega^3 - 2(\Theta_2 + \mathrm{B}_2)\omega\\ = 2(\Theta_2 + \mathrm{B}_2)\omega(2\omega^2 - 1)\end{array} \quad (237)$$

$$\begin{array}{l}P_{I\omega} = \mathrm{B}_1 - 3(\Theta_2 + \mathrm{B}_2)\omega^2 + 5\Theta_5\omega^4\\ Q_{R\omega} = -2(\mathrm{A}_2 + C_2)\omega\\ Q_{I\omega} = (\mathrm{A}_1 + C_1) - 3\mathrm{A}_3\omega^2\end{array} \quad (238)$$

$$\begin{array}{l}P_{R\tau} = 0; P_{I\tau} = 0; Q_{R\tau} = 0\\ Q_{I\tau} = 0; \omega_\tau = -F_\tau/F_\omega\end{array} \quad (239)$$

$$\begin{array}{l}P_{R\omega}P_R = 2(\Theta_2 + \mathrm{B}_2)\omega(2\omega^2 - 1)[(\Theta_2 + \mathrm{B}_2)\omega^4\\ -(\Theta_2 + \mathrm{B}_2)\omega^2] = 2(\Theta_2 + \mathrm{B}_2)\omega(2\omega^2 - 1)(\Theta_2\\ +\mathrm{B}_2)\omega^2[\omega^2 - 1] = 2(\Theta_2 + \mathrm{B}_2)^2\omega^3(2\omega^2\\ -1)[\omega^2 - 1]\end{array} \quad (240)$$

$$\begin{array}{l}P_{R\omega}P_R = 2(\Theta_2 + \mathrm{B}_2)^2\omega^3(2\omega^2 - 1)[\omega^2 - 1]\\ Q_{R\omega}Q_R = -2(\mathrm{A}_2 + C_2)\omega[C_0 - (\mathrm{A}_2 + C_2)\omega^2]\end{array} \quad (241)$$

$$\begin{array}{l}F_\tau = 2[(P_{R\tau}P_R + P_{I\tau}P_I)\\ -(Q_{R\tau}Q_R + Q_{I\tau}Q_I)] = 0\\ P_R P_{I\omega} = (\Theta_2 + \mathrm{B}_2)\omega^2(\omega^2 - 1)[\mathrm{B}_1\\ -3(\Theta_2 + \mathrm{B}_2)\omega^2 + 5\Theta_5\omega^4]\end{array} \quad (242)$$

$$\begin{array}{l}P_I P_{R\omega} = 2\omega^2[B_1 - (\theta_2 + B_2)\omega^2\\ +\theta_5\omega^4](\theta_2 + B_2)(2\omega^2 - 1)\end{array} \quad (243)$$

$$\begin{array}{l}Q_R Q_{I\omega} = [C_0 - (\mathrm{A}_2 + C_2)\omega^2][(\mathrm{A}_1\\ +C_1) - 3\mathrm{A}_3\omega^2]\\ Q_I Q_{R\omega} = -2\omega^2[(\mathrm{A}_1 + C_1) - \mathrm{A}_3\omega^2](\mathrm{A}_2 + C_2)\end{array} \quad (244)$$

$$V = (P_R P_{I\tau} - P_I P_{R\tau}) - (Q_R Q_{I\tau} - Q_I Q_{R\tau}) = 0 \quad (245)$$

$F(\omega, \tau) = 0$. Differentiating with respect to τ and we get

$$\begin{array}{l}F_\omega\frac{\partial\omega}{\partial\tau} + F_\tau = 0; \tau \in I \Rightarrow \frac{\partial\omega}{\partial\tau} = -\frac{F_\tau}{F_\omega}\\ \Lambda^{-1}(\tau) = (\frac{\partial\mathrm{Re}\lambda}{\partial\tau})_{\lambda=i\omega}; \frac{\partial\omega}{\partial\tau} = \omega_\tau = -\frac{F_\tau}{F_\omega}\end{array} \quad (246)$$

$$\begin{array}{l}\Lambda^{-1}(\tau) = \mathrm{Re}\{\frac{-2[U+\tau|P|^2]+iF_\omega}{F_\tau+i2[V+\omega|P|^2]}\}\\ \mathrm{sign}\{\Lambda^{-1}(\tau)\} = \mathrm{sign}\{(\frac{\partial\mathrm{Re}\lambda}{\partial\tau})_{\lambda=i\omega}\}\end{array} \quad (247)$$

$$\mathrm{sign}[\Lambda^{-1}(\tau)] = \mathrm{sign}\{F_\omega\}\mathrm{sign}\{\frac{V + \frac{\partial\omega}{\partial\tau}U}{|P|^2} + \omega + \frac{\partial\omega}{\partial\tau}\tau\} \quad (248)$$

We shall presently examine the possibility of stability transitions (bifurcations) RFID TAG detector system, about the equilibrium point $E^{(0)}(X^{(0)}, Y^{(0)}, I_{L_1}^{(0)}, I_{R_j}^{(0)}, I_{R_S}^{(0)})$. $E^{(0)}(X^{(0)}, Y^{(0)}, I_{L_1}^{(0)}, I_{R_j}^{(0)}, I_{R_S}^{(0)}) = (0,0,0,0,0)$ as a result of a variation of delay parameter τ. The analysis consists in identifying the roots of our system characteristic equation situated on the imaginary axis of the complex λ-plane Where by increasing the delay parameter τ, Re λ may at the crossing, changes its sign from - to +, i.e. from a stable focus $E^{(*)}$ to an unstable one, or vice versa. This feature may be further assessed by examining the sign of the partial derivatives with respect to τ.

$$\Lambda^{-1}(\tau) = (\frac{\partial \mathrm{Re}\lambda}{\partial\tau})_{\lambda=i\omega} \quad (249)$$

$$\begin{gathered}\Lambda^{-1}(\tau) = (\frac{\partial \mathrm{Re}\lambda}{\partial\tau})_{\lambda=i\omega} \\ L_P, L_1, C_f, R_{in}, R_S, C_P, R_j, ... = const; \omega \in R_+\end{gathered} \quad (250)$$

$$U = (P_R P_{I\omega} - P_I P_{R\omega}) - (Q_R Q_{I\omega} - Q_I Q_{R\omega}) \quad (251)$$

$$\begin{aligned} U &= (P_R P_{I\omega} - P_I P_{R\omega}) - (Q_R Q_{I\omega} - Q_I Q_{R\omega}) \\ &= (\Theta_2 + \mathrm{B}_2)\omega^2(\omega^2 - 1)[\mathrm{B}_1 - 3(\Theta_2 + \mathrm{B}_2)\omega^2 + 5\Theta_5\omega^4] \\ &- 2\omega^2[\mathrm{B}_1 - (\Theta_2 + \mathrm{B}_2)\omega^2 + \Theta_5\omega^4](\Theta_2 + \mathrm{B}_2)(2\omega^2 - 1) \\ &- [C_0 - (\mathrm{A}_2 + C_2)\omega^2][(\mathrm{A}_1 + C_1) - 3\mathrm{A}_3\omega^2] \\ &- 2\omega^2[(\mathrm{A}_1 + C_1) - \mathrm{A}_3\omega^2](\mathrm{A}_2 + C_2) \end{aligned} \quad (252)$$

The single diode detector, R_L is the video load resistance which not seen in RFID TAG receiver detector equivalent circuit. L_1, the shunt inductance, provides a current return path for the diode, and is chosen to be large compared to diode impedance at the input or RF frequency. C_1, the bypass capacitance, is chosen to be sufficiently large that is capacitive reactance is small compared to the diode impedance, but small enough to avoid having it resistance load the video circuit. P_{in} is the RF input power applied to the detector circuit and V_0 is the output voltage appearing across R_L. L_P is packaged parasitic inductance (Schottky linear equivalent circuit). C_P is package parasitic capacitance. R_S is the diode's parasitic series resistance. C_j is junction parasitic capacitance, and R_j is the diode's junction resistance. L_P, C_P, and R_L are constants. R_S has some small variation with temperature, but that variation is not a significant parameter in this analysis. C_j is a function of both temperature and DC bias, but this analysis concerns itself with the zero bias detectors and the variation with temperature is not significant. R_j is a key element in equivalent circuit – its behaviour clearly will affect the performance of the detector circuit. For our stability switching analysis, we choose typical Schottky detector parameter values: L_P=2 nH, R_S=1.5 ohm, C_P=0.08 pF, C_j=0.2 pF, R_j=500 ohm, R_L=100 Kohm, R_{in}=1 kohm, L_1=1 mH, C_1=1 uF.

$$\begin{gathered}\sigma_1 = -5 \times 10^{11}; \sigma_2 = -6.2492 \times 10^{21} \\ \sigma_3 = 5 \times 10^{17}; \sigma_4 = 6.25 \times 10^{21} \\ \sigma_5 = -10^6; \sigma_6 = 10^{10}\end{gathered} \quad (253)$$

$$\begin{gathered}\sigma_7 = 8.33 \times 10^{12}; \sigma_8 = 3.33 \times 10^{12} \\ \sigma_9 = -1.155 \times 10^{13}; \psi_1 = -10^{16} \\ \psi_2 = -1.0001 \times 10^{10}\end{gathered} \quad (254)$$

$$\begin{gathered}\psi_3 = -8.22 \times 10^{28}; \psi_4 = -8.2212 \times 10^{22} \\ \psi_5 = -8.22 \times 10^{22}; \Theta_2 = -5 \times 10^{27} \\ \Theta_3 = -5.0005 \times 10^{21}\end{gathered} \quad (255)$$

$$\begin{gathered}\Theta_4 = -5.1 \times 10^{11}; \Theta_5 = -1 \\ \mathrm{A}_1 = -6.2497 \times 10^{37}; \mathrm{A}_2 = -6.2498 \times 10^{31} \\ \mathrm{A}_3 = -6.2492 \times 10^{21}\end{gathered} \quad (256)$$

$$\begin{gathered}\mathrm{B}_1 = -4.11 \times 10^{40}; \mathrm{B}_2 = -4.1106 \times 10^{34} \\ \mathrm{B}_3 = -5.8572 \times 10^{24}; \mathrm{B}_4 = -1.155 \times 10^{13}\end{gathered} \quad (257)$$

$$\begin{gathered}\mathrm{C}_0 = 6.8997 \times 10^{48}; \mathrm{C}_1 = 6.9178 \times 10^{42} \\ \mathrm{C}_2 = -2.0116 \times 10^{34}; \Pi_0 = -4.7606 \times 10^{97}\end{gathered} \quad (258)$$

$$\begin{gathered}\Pi_2 = -4.8132 \times 10^{85}; \Pi_4 = -3.3789 \times 10^{75} \\ \Pi_6 = -1.6897 \times 10^{69}; \Pi_8 = 1.6897 \times 10^{69} \\ \Pi_{10} = 1\end{gathered} \quad (259)$$

Then we get the expression for $F(\omega, \tau)$ Schottky diode detector parameter values. We find those ω, τ values which fulfill $F(\omega, \tau) = 0$. We ignore negative, complex, and imaginary values of ω for specific τ values. $\tau \in [0.001...10]$, we can be express by 3D function $F(\omega, \tau) = 0$. We plot the stability switch diagram based on different delay values of our Schottky diode detector.

$$\begin{aligned}\Lambda^{-1}(\tau) &= (\tfrac{\partial \mathrm{Re}\lambda}{\partial\tau})_{\lambda=i\omega} \\ &= \mathrm{Re}\{\tfrac{-2[U+\tau|P|^2]+iF_\omega}{F_\tau+2i[V+\omega|P|^2]}\}\end{aligned} \quad (260)$$

$$\begin{aligned}\Lambda^{-1}(\tau) &= (\tfrac{\partial \mathrm{Re}\lambda}{\partial\tau})_{\lambda=i\omega} \\ &= \tfrac{2\{F_\omega(V+\omega P^2) - F_\tau(U+\tau P^2)\}}{F_\tau^2 + 4(V+\omega P^2)^2}\end{aligned} \quad (261)$$

The stability switch occurs only on those delay values (τ) which fit the equation: $\tau = \frac{\theta_+(\tau)}{\omega_+(\tau)}$ and $\theta_+(\tau)$ is the solution of $\sin\theta(\tau) = ...; \cos\theta(\tau) = ...$ when $\omega = \omega_+(\tau)$ if only ω_+ is feasible. Additionally, when all Schottky diode detectors' parameters are known and the stability switch due to various time delay values τ is described in the following expression:

$$\begin{gathered}\mathrm{sign}\{\Lambda^{-1}(\tau)\} = \mathrm{sign}\{F_\omega(\omega(\tau),\tau)\}\mathrm{sign}\{\tau\omega_\tau(\omega(\tau)) \\ +\omega(\tau) + \tfrac{U(\omega(\tau))\omega_\tau(\omega(\tau)) + V(\omega(\tau))}{|P(\omega(\tau))|^2}\}\end{gathered} \quad (262)$$

Remark: we know $F(\omega, \tau) = 0$ implies its roots $\omega_i(\tau)$ and finding those delays values τ which ω_i is feasible. There are τ values which give complex ω_i or imaginary number, then unable to analyse stability [4] [5]. F function is independent on τ the parameter $F(\omega, \tau) = 0$.

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The results: We find those ω, τ values which fulfill $F(\omega, \tau) = 0$. We ignore negative, complex, and imaginary values of ω. We define new MATLAB script parameters: $\pi_{2k} \rightarrow G_{2k}$(k=0...5). Running a MATLAB script to find ω values, gives the following results:

$$F(\omega) = 0 \Rightarrow \omega_1 = 1.0\mathrm{e} + 034*;\omega_2 = 0 + 4.1106\mathrm{i} \quad \omega_3 = 0 - 4.1106\mathrm{i};\omega_4, ..., \omega_{11} = 0 \tag{263}$$

Next is to find those ω, τ values which fulfil $\sin\theta(\tau) = ...$;

$$\sin(\omega\tau) = \frac{-P_R Q_I + P_I Q_R}{|Q|^2}; \cos\theta(\tau) = ... \tag{264}$$

$$\cos(\omega\tau) = -\frac{(P_R Q_R + P_I Q_I)}{|Q|^2}; |Q|^2 = Q_R^2 + Q_I^2 \tag{265}$$

Finally, we plot the stability switch diagram

$$g(\tau) = \Lambda^{-1}(\tau) = (\frac{\partial \mathrm{Re}\lambda}{\partial \tau})_{\lambda = i\omega} \tag{266}$$

$$g(\tau) = \Lambda^{-1}(\tau) = (\frac{\partial \mathrm{Re}\lambda}{\partial \tau})_{\lambda = i\omega} = \frac{2\{F_\omega(V+\omega P^2) - F_\tau(U+\tau P^2)\}}{F_\tau^2 + 4(V+\omega P^2)^2} \tag{267}$$

$$\mathrm{sign}[g(\tau)] = \mathrm{sign}[\Lambda^{-1}(\tau)] = \mathrm{sign}[(\frac{\partial \mathrm{Re}\lambda}{\partial \tau})_{\lambda = i\omega}] = \mathrm{sign}[\frac{2\{F_\omega(V+\omega P^2) - F_\tau(U+\tau P^2)\}}{F_\tau{}^2 + 4(V+\omega P^2)^2}] \tag{268}$$

$$F_\tau^2 + 4(V + \omega P^2)^2 > 0 \tag{269}$$

$$\mathrm{sign}[\Lambda^{-1}(\tau)] = \mathrm{sign}\{F_\omega(V + \omega P^2) - F_\tau(U + \tau P^2)\} \tag{270}$$

$$\mathrm{sign}[\Lambda^{-1}(\tau)] = \mathrm{sign}\{[F_\omega][(V + \omega P^2) - \frac{F_\tau}{F_\omega}(U + \tau P^2)]\} \quad \omega_\tau = -\frac{F_\tau}{F_\omega}; \omega_\tau = (\frac{\partial \omega}{\partial \tau})^{-1} = -\frac{\partial F/\partial \omega}{\partial F/\partial \tau} \tag{271}$$

$$\mathrm{sign}[\Lambda^{-1}(\tau)] = \mathrm{sign}\{[F_\omega][V + \omega_\tau U + \omega P^2 + \omega_\tau \tau P^2]\} \tag{272}$$

$$\mathrm{sign}[\Lambda^{-1}(\tau)] = \mathrm{sign}\{[F_\omega][\frac{1}{P^2}][\frac{V + \omega_\tau U}{P^2} + \omega + \omega_\tau \tau]\} \tag{273}$$

$$\mathrm{sign}[\frac{1}{P^2}] > 0 \Rightarrow \mathrm{sign}[\Lambda^{-1}(\tau)] = \mathrm{sign}\{[F_\omega][\frac{V+\omega_\tau U}{P^2} + \omega + \omega_\tau \tau]\} \tag{274}$$

$$\mathrm{sign}[\Lambda^{-1}(\tau)] = \mathrm{sign}[F_\omega]\mathrm{sign}[\frac{V+\omega_\tau U}{P^2} + \omega + \omega_\tau \tau] \quad F_\omega = 2[(P_{R\omega}P_R + P_{I\omega}P_I) - (Q_{R\omega}Q_R + Q_{I\omega}Q_I)] \tag{275}$$

We check the sign of $\Lambda^{-1}(\tau)$ according the following rule:

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sign$[F_\omega]$	sign$[\frac{V+\omega_\tau U}{P^2} + \omega + \omega_\tau \tau]$	sign$[\Lambda^{-1}(\tau)]$
+/-	+/-	+
+/-	-/+	-

.

Table 3. RFID TAG receiver detector system sign of $\Lambda^{-1}(\tau)$

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If sign$[\Lambda^{-1}(\tau)] > 0$ then the crossing proceeds from (-) to (+) respectively (stable to unstable). If sign$[\Lambda^{-1}(\tau)] < 0$ then the crossing proceeds from (+) to (-) respectively (unstable to stable). Anyway the stability switching can occur only for $\omega = 1.0e + 034$ or ω=0.

V. Conclusion

RFID TAGs detector circuit is characterized by delay elements in time which can influence RFID TAGs detector stability in time. There are two main RFID TAGs detector variables which are affected by Schottky parasitic in time, current flows through Schottky diode's package parasitic inductance (L_p) and the current flows through Schottky diode's parasitic resistance (R_s). The two time delays (τ_1, τ_2) are not the same but can be categorized to some subcases due to delay elements in time. The first case is when there is RFID TAGs detector Schottky diode's package parasitic inductance's current delay in time but no delay on Schottky diode's parasitic resistance's current delay in time. The second case is when there is no RFID TAGs detector Schottky diode's package parasitic inductance's current delay in time but there is a delay in time on Schottky diode's parasitic resistance's current. The third case is when both RFID TAGs detector Schottky diode's package parasitic inductance's current delay in time and a delay in time on Schottky diode's parasitic resistance's current exist. For simplicity of our analysis we consider in the third case two delays are the same (there is a difference but it is neglected in our analysis). In each case we derive the related characteristic equation. The characteristic equation is dependent on RFID TAGs detector overall parameters and delay elements in time. Upon mathematics manipulation and [BK] theorems and definitions we derive the expression which gives us clear picture on RFID TAGs detector stability map. The stability map gives all possible options for stability segments, each segment belong to different time delay values segment. RFID TAGs detector stability analysis can be influence either by RFID TAGs detector overall parameters values. We left this analysis and do not discuss it in the current article.

VI. Appendix A

A. *Appendix A_1 (Lemma 1.1)*

Assume that $\omega(\tau)$ is a positive and real root of $F(\omega, \tau) = 0$ and defined for $\tau \in I$, which is continuous and differentiable. Assume further that if $\lambda = i\omega; \omega \in R$, then $P_n(i\omega, \tau) + Q_n(i\omega, \tau) \neq 0, \tau \in \mathrm{R}$ hold true. Then the functions $S_n(\tau), n \in N_0$, are continuous and differentiable on I.

B. *Appendix A_2 (Theorem 1.2)*

Assume that $\omega(\tau)$ is a positive real root of $F(\omega, \tau) = 0$ defined for $\tau \in I, I \subseteq R_{+0}$, and at some $\tau^* \in I$, $S_n(\tau^*) = 0$. For some $n \in N_0$ then a pair of simple conjugate pure imaginary roots $\lambda_+(\tau^*) = i\omega(\tau^*)$ and $\lambda_-(\tau^*) = -i\omega(\tau^*)$ of $D(\lambda, \tau) = 0$ exist at $\tau = \tau^*$ which crosses the imaginary axis from left to right if $\delta(\tau^*) > 0$ and cross the imaginary axis from right to left if $\delta(\tau^*) < 0$ where

$$\delta(\tau^*) = \mathrm{sign}\{\frac{d\mathrm{Re}\lambda}{d\tau}|_{\lambda = i\omega(\tau^*)}\} = \mathrm{sign}\{F_\omega(\omega(\tau^*), \tau^*)\}\mathrm{sign}\{\frac{dS_n(\tau)}{d\tau}|_{\tau=\tau^*}\} \tag{276}$$

The theorem becomes

$$\begin{aligned}&\text{sign}\{\tfrac{d\text{Re}\lambda}{d\tau}|_{\lambda=i\omega\pm}\}\\&=\text{sign}\{\pm\Delta^{1/2}\}\text{sign}\{\tfrac{dS_n(\tau)}{d\tau}|_{\tau=\tau^*}\}\end{aligned} \quad (277)$$

C. Appendix A_3 (Theorem 1.3)

The characteristic equation has a pair of simple and conjugate pure imaginary roots $\lambda = \pm\omega(\tau^*), \omega(\tau^*)$ real at $\tau^* \in I$ if $S_n(\tau^*) = \tau^* - \tau_n(\tau^*) = 0$ for some $n \in N_0$. If $\omega(\tau^*) = \omega_+(\tau^*)$, this pair of simple conjugate pure imaginary roots crosses the imaginary axis from left to right if $\delta_+(\tau^*) > 0$ and crosses the imaginary axis from right to left if $\delta_+(\tau^*) < 0$ where

$$\begin{aligned}&\delta_+(\tau^*) = \text{sign}\{\tfrac{d\text{Re}\lambda}{d\tau}|_{\lambda=i\omega_+(\tau^*)}\}\\&=\text{sign}\{\tfrac{dS_n(\tau)}{d\tau}|_{\tau=\tau^*}\}\end{aligned} \quad (278)$$

If $\omega(\tau^*) = \omega_-(\tau^*)$, this pair of simple conjugate pure imaginary roots cross the imaginary axis from left to right if $\delta_-(\tau^*) > 0$ and crosses the imaginary axis from right to left If $\delta_-(\tau^*) < 0$ where

$$\begin{aligned}&\delta_-(\tau^*) = \text{sign}\{\tfrac{d\text{Re}\lambda}{d\tau}|_{\lambda=i\omega_-(\tau^*)}\}\\&=-\text{sign}\{\tfrac{dS_n(\tau)}{d\tau}|_{\tau=\tau^*}\}\end{aligned} \quad (279)$$

If $\omega_+(\tau^*) = \omega_-(\tau^*) = \omega(\tau^*)$ then $\Delta(\tau^*) = 0$ and $\text{sign}\{\frac{d\text{Re}\lambda}{d\tau}|_{\lambda=i\omega(\tau^*)}\} = 0$ and the same is true when $S_n(\tau^*) = 0$. The following result can be useful in identifying values of τ where stability switches happened.

D. Appendix A_4 (Theorem 1.4)

Assume that for all $\tau \in I$, $\omega(\tau)$ is defined as a solution of $F(\omega, \tau) = 0$ then $\delta_\pm(\tau) = \text{sign}\{\pm\Delta^{1/2}(\tau)\}\text{sign}D_\pm(\tau)$.

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